Longevity Risk, Health Status and Annuity Pricing

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UNSW-CSIRO workshop
Risk: Modelling, Optimization, Inference - with Applications to Finance, Insurance and Superannuation
11-12 December 2014, UNSW Australia, Sydney

Research aims

- Assess impact of health heterogeneity on annuity pricing and optimal annuitization decisions
- Develop multiple state model that includes:
  - heterogeneity in health status
  - systematic improvement trends and uncertainty
- Calibrate to (Australian) population level health (chronic disease) and cohort mortality data
- Quantify impact on annuity pricing and compare to other models
- Work in progress - analysis of annuitization with health status (when, how much, immediate versus deferred)

Background

- Observed heterogeneity: individual data, GLMM
  - Meyricke and Sherris (2013)
- Unobserved heterogeneity: multi-state Markov models
  - Levinson (1959), Redington (1969)
  - Pollard (1970)
  - Le Bras (1976) (equivalent to Vaupel's (1979) frailty model)
- Markov ageing models
  - Lin and Liu (2007), Su and Sherris (2012): physiological ages
  - Liu and Lin (2013): subordinated using time-change
  - Zhou and Sherris (2014): time-change, calibrate to health status, apply to an annuity fund to assess selection and heterogeneity

Markov Ageing Model

- Ageing process modeled in terms of changes in physiological functions
- Physiological age: represents the degree of ageing
    - For any given age there is a range of physiological ages (representing heterogeneity)
    - Higher mortality rates for higher physiological ages

Transition matrix - Lin and Liu 2007

- Model based on "physiological age"
- 200 transient states and 1 absorbing state (death)
- Phase-type distribution for time to death
- Transition matrix:

$$\Lambda = \begin{pmatrix} -(\lambda_1 + q_1) & \lambda_1 & 0 & \cdots & 0 \\ 0 & -(\lambda_2 + q_2) & \lambda_2 & \cdots & 0 \\ 0 & 0 & -(\lambda_3 + q_3) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -q_n \end{pmatrix}$$

- λ_i is constant after the fourth state
- $q_i = i^p \cdot q + h_i$, where p and q are constants, h_i has two values: higher during the accident hump ages.

Markov Ageing Model - Su and Sherris (2012)

- Markov process with n (=100) transient states and 1 absorbing death state, describing the aging process of human beings

$$\mathbf{\Lambda} = \begin{pmatrix} -(\lambda_1+q_1) & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & -(\lambda_k+q_k) & \lambda_k & \cdots & 0 \\ 0 & \cdots & 0 & -(\lambda+\gamma+\alpha e^{\beta(k+1)}) & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & -(\alpha+e^{\beta n}) \end{pmatrix}$$

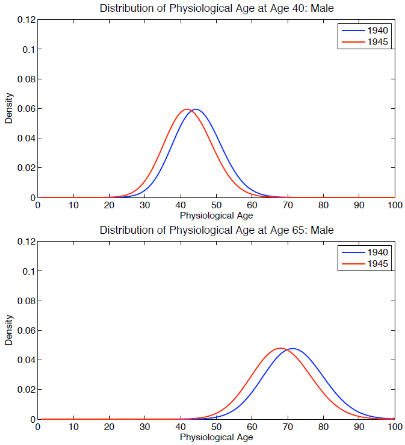
- 4 developmental periods
- $\lambda_i = \lambda$ for $i = 5, 6, ..., n-1$
- Death rates for $i = 5, 6, ...n$

$$q_i = \begin{cases} \gamma + \gamma_1 + \alpha e^{\beta i} & : \text{ for } i_1 < i < i_2 \\ \gamma + \alpha e^{\beta i} & : \text{ otherwise} \end{cases}$$

- Time to death follows phase-type distribution with $\hat{S}(t) = \alpha exp(\Lambda t)e$
- $\hat{q}_x = \frac{\hat{S}_x - \hat{S}_{x+1}}{\hat{S}_x}$
- Weighted least squares estimation: $\sum_x (q_x - \hat{q}_x)^2 \cdot w_x$

Distribution of physiological ages - Su and Sherris (2012)

Increased heterogeneity at older ages



Systematic Mortality Risk - Lin and Liu (2013) and Zhou and Sherris (2014)

- Underlying multi-state model, made stochastic through time-change
- 5 transient states
- Gamma time-change:
  - Survival probability at time t = survival probability given by underlying model at time γ_t, $S(\gamma_t)$, $\gamma()$ is a Gamma process
- Zhou and Sherris (2014) calibrate to both health status and survival probabilities

Model

- Transition rates parameterised by: time (s), cohort (c), and a Gamma time-change
- 5 transient states (health states) with transitions: $\lambda_i(s) = n_i + k \cdot \exp(m \cdot s)$
- In state i, transition to absorbing state at time s: $q_{c,i}(s) = d_i r^c \cdot \exp(b \cdot s)$
  - d_i: proportional relationship between states, $d_1 \leq d_2 \leq d_3 \leq d_4 \leq d_5$
  - r^c: cohort trend: r is a positive constant less than 1, c is cohort number ($c = 1$ for 1935, ... $c = 39$ for 1973)
  - e^{bs}: exponential increase with time s
- Gamma time-change: survival probability at time t = survival probability given by underlying model at time γ_t, which is Gamma distributed with mean t and variance νt.

Survival Probabilities

- For each cohort c, the survival probability in t year's time is:

$$S_c(t) = \pi_{c,0} \exp\left(\sum_{s=0}^{t-1} \Lambda_c(s)\right) \mathbf{1}$$

- Density function of life time distribution of cohort c, at time t:

$$f_c(t) = \pi_{c,0} \exp\left(\sum_{s=0}^{t-1} \Lambda_c(s)\right) \cdot (-\Lambda_c(t-1) \cdot \mathbf{1})$$

Survival Probabilities

- The expected value of $S_c(t)$ is:

$$\mathrm{E}\left(S_c(t)\right) = \pi_{c,0} \exp\left(\sum_{s=0}^{t-1} \tilde{\Lambda}_c(s)\right) \mathbf{1} \tag{1}$$

where

$$\tilde{\Lambda}_c(s) = \sum_{i=1}^{5} \left[\frac{1}{\nu} \ln\left(1 + \nu\left(-q_{c,i}(s) - \lambda_i(s)\right)\right)\right] \mathbf{h_i v_i}$$

$\mathbf{h_i}$ and $\mathbf{v_i}$ are the right and left eigenvectors of the transition matrix Λ_c and such that $v_i h_i = 1$

Health and Mortality Data

The model was calibrated to survival and health data for Australian cohorts 1934-1973 (male and female combined). The following data sources were used:

- cohort death rates: Human Mortality Database
- prevalence of health conditions: National Health Survey 2007-2008, Ritchie (1992) estimated average dementia prevalence, Australian Cancer Incidence and Mortality Books 2008
- death by cause: WHO mortality database, for Australia 2006 (number of deaths by age), Australian Bureau of Statistics Causes of Death 2008 (aggregate of all ages)
- population size (number of persons aged x in a specific year): Human Mortality Database

Health and Mortality Data

- The health conditions were ranked and divided into five groups according to their probability of causing death for 65-74 year-old individuals within 1 year.
  - Mortality by condition: number of deaths caused divided by prevalence of the condition.
  - Deaths by cause data (of 2006) was scaled by the ratio of total number of deaths to match the prevalence data (of 2008).
- Long term conditions are assumed to be independent and for a person affected by more than one condition, the highest death rate among all of the conditions was used for the death rate.

Health Conditions and States

- State 1 includes - hypotension, mood disorder.
- State 2 includes - asthma, hypertensive disorders, endocrine disorders.
- State 3 includes - dementia, diabetes, substance abuse.
- State 4 includes - breast cancer, hodgkins disease, prostate cancer.
- State 5 includes - bladder cancer, kidney cancer, liver cancer, lung cancer.
    - Prevalence varies by age group
    - Mortality rates similar across ages by health condition

Health States and Mortality Rates

To fit the model, require

- expected health state distribution and
- expected mortality rates ${}_1q_x$ for persons aged 35 to 75 in 2008.

Data gives us

- the average health state distribution and average ${}_1q_x$ in the data for the expected values for persons aged: 29.5, 39.5, 49.5, 59.5

Values for individual ages are estimated using interpolation.

Model Calibration

- Parameters were estimated by minimizing the difference between observed and model values of
  1. expected cohort survival probabilities,
  2. health state-distribution in 2008 and
  3. probability of death within a year for each state in 2008.
- Calibration was for both the parameters in the transition matrix and the initial state distribution of each cohort at age 30.

Health States

Increased proportion in better health states for later cohorts

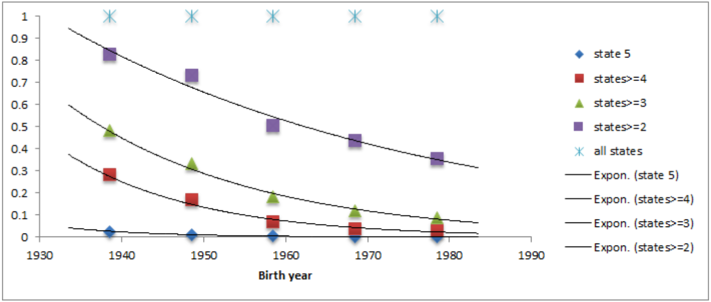


Figure : Health state distribution y values are percentage of people in a specified health state or worse. Smoothed using an exponential trend line of the form $y = a \cdot \exp(bx)$

Mortality rates by state

Improvement in mortality rates for less healthy states for later cohorts

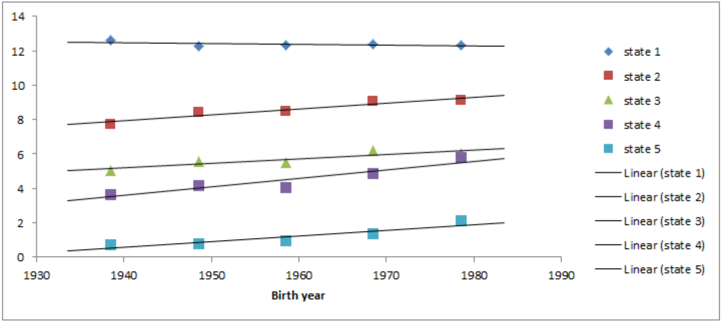


Figure : Mortality rates ${}_1q_x$ by cohort. y values are: $-\ln({}_1q_x)$. Interpolated using linear trend line of the form $y = ax + b$

Parameter estimates

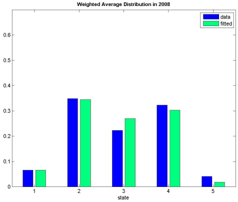
- $\lambda_i(s) = n_i + k \cdot \exp(m \cdot s)$ transition rates
- mortality parameters: r allows for cohort effect; b allows for age effect; d_i captures relative mortality rate across states

	Fitted values	%change in a_{65} -10%	+10%
m	0.06949	2.94%	-3.76%
k	0.00264	0.01%	-0.01%
n_1	4.85E-07	0.00%	0.00%
n_2	0.00732	0.04%	-0.04%
n_3	0.03008	0.06%	-0.06%
n_4	0.00929	0.02%	-0.02%

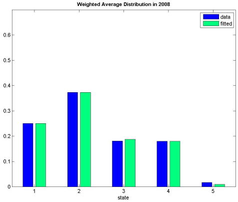
Parameter estimates

	Fitted values	%change in a_{65} -10%	%change in a_{65} +10%
d_1	8.87E-07	0.00%	0.00%
d_2	0.00006	0.00%	0.00%
d_3	0.00115	0.04%	-0.03%
d_4	0.00154	0.05%	-0.05%
d_5	0.26955	0.10%	-0.09%
r	0.98802	1.96%	-1.81%
b	0.03200	0.17%	-0.17%
ν	0.02937	0.00%	0.00%

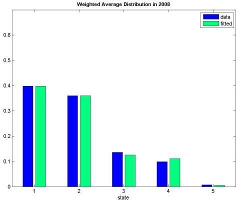
Fitted health states distribution



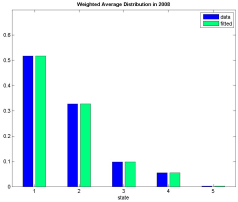
Average of cohorts 1934-1943



Average of cohorts 1944-1953

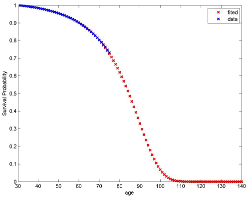


Average of cohorts 1954-1963

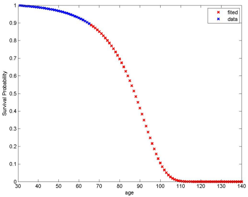


Average of cohorts 1964-1973

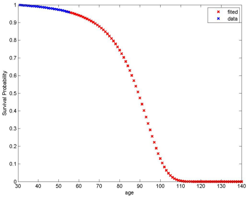
Fitted survival probabilities and extrapolation



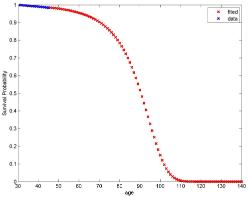
Cohort 1934



Cohort 1944

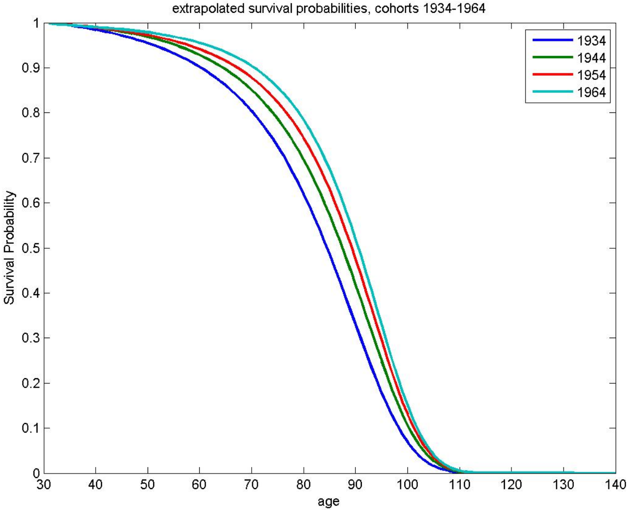


Cohort 1954

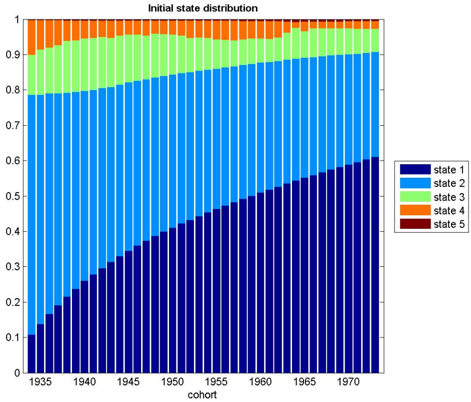


Cohort 1964

Survival Probabilities



Initial State Distributions by Cohort



Fitted initial states distribution (cohorts 1934-1973 at age 30) - calibrated to 2008 health states by ten year age groups

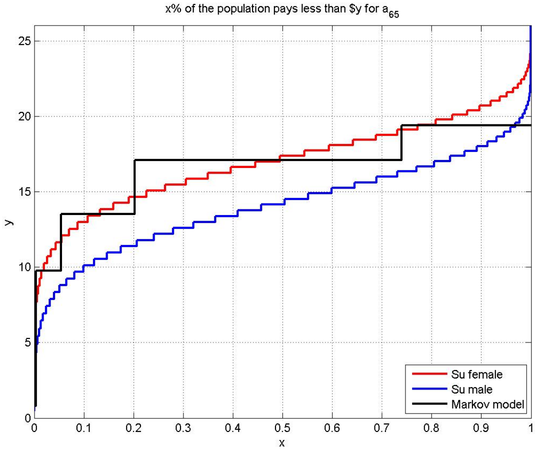
Annuity valuation

To determine expected annuity values by health states the following assumptions are used:

- maximum age = 120
- fixed annual interest rate = 3%
- no allowance for idiosyncratic risk.
- 1948 cohort - aged 60 in 2008 and 65 in 2013

Aggregate survival rates are simulated for given initial states.

Comparison with Su-Sherris model - cohort 1940



Annuity risk loadings - 20% mortality shock

- Mortality rates are reduced by 20% (all transition rates including death are multiplied by 0.8)
- Expected values and standard deviations of a_{65} (cohort 1948), $\nu = 0.1$

	original q		$0.8 \times q$
	mean	s.d.	mean
State 1	19.42	0.16	+5.7%
State 2	17.13	0.18	+6.7%
State 3	13.62	0.21	+8.1%
State 4	9.84	0.21	+10.7%
State 5	0.89	0.12	+32.3%
Average	15.78	0.17	+7.9%

- Mortality rate reduction by 20% in Australian Government Actuary Lifetable 2005-07 results in an increase of annuity values by 7.2% for males and 5.7% for females.

Annuity returns - alive state

Uncertain returns if alive depending on health status
Mortality credit reduced by loss in annuity value if health deteriorates

	a_{45}	a_{65}	a_{75}	a_{85}	a_{95}	$_{20\|}a_{45}$	$_{20\|}a_{65}$
state 1	25.10	19.42	15.80	11.95	8.26	10.23	4.71
state 2	23.04	17.13	13.58	9.97	6.68	8.26	3.00
state 3	19.23	13.62	10.48	7.43	4.81	5.15	1.32
state 4	15.49	9.84	7.06	4.65	2.78	3.06	0.50
state 5	1.89	0.89	0.57	0.34	0.19	0.00	0.00
average	22.53	15.78	11.65	7.49	4.03	8.07	2.74

Table : Expected values

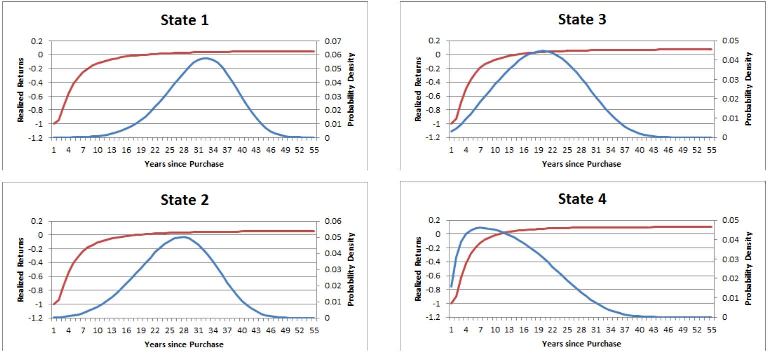
Annuity realized returns

- Return on purchasing life annuity for a_{65} at age 65 for states 1 to 5.
- $RR_t =$ realized return if die in year t aged $65 + t$.
- Average RR is calculated as RR_t by $\sum_{t=1}^{55} RR_x \cdot Prob(t \leq T < t+1)$

	State 1	State 2	State 3	State 4	State 5	Ave*
$RR_t > 3\%,\ t \geq \ldots$	30	25	18	12	1	22
$\mathrm{E}\left[T(x)\right]$	30.29	25.31	18.75	12.77	0.94	23.22
Maximum RR	0.0475	0.0554	0.0718	0.1011	1.1248	0.0609
Average RR	0.0261	0.0186	-0.0102	-0.0831	-0.2618	-0.0171
Variance of RR	0.0005	0.0030	0.0187	0.0658	0.6337	0.0279

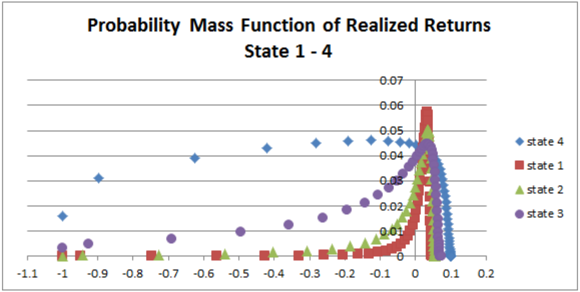
*: average of persons aged 65 in the population

Realized returns (red) and Probability Density of Death (blue)



Probability Density of Realized Returns

From an individual perspective there is considerable downside risk in returns on life annuities allowing for health states.



Summary

- Presented a multiple state model with health status for annuity pricing
- Includes improvement by cohort, Gompertz age profile for mortality
- Systematic mortality risk from subordinated Gamma process
- Model calibrated to population level health condition prevalence and (by cause) mortality rate data as well as cohort mortality data
- Assessed impact on annuity returns - implications for individual optimal annuitization
- Further work

Thank you for your attention

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Acknowledgement: ARC Linkage Grant Project LP0883398 Managing Risk with Insurance and Superannuation as Individuals Age with industry partners PwC, APRA and the World Bank as well as the support of the Australian Research Council Centre of Excellence in Population Ageing Research project CE110001029.

ASTIN and AFIR/ERM 2015

2015 ASTIN and AFIR/ERM Colloquium of the International Actuarial Association will be held in Sydney, Australia 23 to 27 August 2015.

Call for Papers
Synopsis Deadline: 27 February 2015
Draft Program Released: 27 March 2015
Paper Submission Deadline: 15 April 2015
Acceptance Deadline: 15 May 2015
Web site:
http://www.actuaries.org/sydney2015/callforpapers.cfm

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