ABSTRACTS

Xuan Duong: *Weighted bounds for singular operators with non-smooth kernels.*

Let T be a multilinear integral operator which is bounded on certain products of Lebesgue spaces on $\mathbb{R}^n$. We assume that its associated kernel satisfies some mild regularity condition which is weaker than the usual Hölder continuity of those in the class of multilinear Calderón-Zygmund singular integral operators. In this talk, given a suitable multiple weight $\vec{w}$, we obtain the bound for the weighted norm of multilinear operators T in terms of $\vec{w}$. As applications, we use this result to obtain the weighted bounds for certain singular integral operators such as linear and multilinear Fourier multipliers and the Riesz transforms associated to Schrödinger operators on $\mathbb{R}^n$. This is joint work with The Anh Bui, José M. Conde-Alonso and Mahdi Hormozi.

Michael Eastwood: *Parabolic geometry in five dimensions.*

There are various interesting non-Riemannian geometries in five dimensions, all based on homogenous spaces of the form G/P for G simple and P parabolic. Starting from scratch, I shall explain how to define them, how they naturally arise, and discuss their key properties.

Tom ter Elst: *Subelliptic operators on Lie groups.*

We review the theory of subelliptic operators on Lie groups and show that these operators share many properties of strongly elliptic operators. Nevertheless, subelliptic operators are usually degenerate at every point of the Lie group.

Rod Gover: *The geometry of CR submanifolds in CR manifolds.*

The problem of understanding CR geometries embedded as submanifolds in higher dimensional CR manifolds arises in higher dimensional complex analysis, including the study of singularities of analytic varieties. It has also been studied intensively in connection with rigidity questions. Despite considerable earlier work the local theory has not been fully understood.

We develop from scratch a CR invariant local theory based on CR tractor calculus (i.e. the associated bundle.) This produces the tools for constructing local invariants and invariant operators in a way parallel to the classical Gauss-Codazzi-Ricci calculus for Riemannian submanifolds. It also enables a practical and conceptual approach to a Bonnet Theorem and potentially the rigidity questions. The study also provides a nice template for developing related submanifold theory in other situations.

This is joint work with Sean Curry.

Enrico Le Donne: *Isometries of nilpotent metric groups.*

Thomas Leistner: *Exceptional conformal structures, ambient metrics and holonomy.*

In the talk we will first review the ambient metric construction of Fefferman and Graham which assigns to a conformal class of metrics a pseudo-Riemannian metric that is Ricci-flat (up to a certain order). We will then relate this to the normal conformal Cartan connection and its holonomy. Finally we will give examples of conformal structures with explicit ambient metrics, including those that are defined by non integrable distributions in dimensions 5 and 6.

Ji Li: *Multi-parameter singular integrals associated with Zygmund dilations and related problems.*

The theory of CalderònZygmund operators plays an important role in modern harmonic analysis. The core of this theory is that the regularity and cancellation conditions are invariant with respect to the one-parameter family of dilations on R^n defined by

$$\delta(x_1, x_2, \cdots, x_n) = (\delta x_1, \cdots, \delta x_n), \quad \delta > 0,$$

in the sense that the kernel $\delta^n K(\delta x)$ satisfies the same conditions with the same bound as $K(x)$. Indeed, the classical singular integrals, maximal functions, A_p weights and multipliers are invariant with respect to such one-parameter dilations.

On the other hand, the multiparameter theory on $\mathbb{R}^n$ began with Zygmund's study of the strong maximal function, and later has been extensively studied by Ricci, Stein, Fefferman, Pipher, and others. It was point out by Fefferman and Pipher that the consideration of these operators associated with Zygmund dilations is a natural next step or the simplest case after those of the classical CalderònZygmund theory and the product space theory.

In this talk we provide our recent study on these multi-parameter singular integral operators which commute with Zygmund dilations. We introduce a class of singular integral operators associated with the Zygmund's dilations and show the boundedness of these operators on L^p for $1 < p < \infty$, which cover those concrete examples of such operators studied by RicciStein, FeffermanPipher and NagelWainger. We further establish the weighted Hardy and BMO spaces associated with the Zygmund's dilations and obtain the end point estimates of these singular integrals.

Gerd Schmalz: *Sasaki manifolds, CR-manifolds and shear-free congruences.*

Adam Sikora: *Spectral analysis of Grushin operators.*

We investigate Riesz transform, spectral multipliers and Bochner-Riesz type results for a class of degenerate elliptic operators which is a natural generalisation of the notion of the classical Grushin operator $\partial_x^2 + x^2\partial_y^2$.

Stephan Tillmann: *Projective structures on manifolds.*

A strictly convex orbifold is the quotient of a strictly convex domain in projective space by a discrete group of projective transformations which preserve the domain. The Hilbert metric is a Finsler metric on the domain that is preserved by the group. In the case the domain is an ellipsoid this is the hyperbolic metric. This talk will survey joint work with Daryl Cooper and Darren Long on strictly convex orbifold of finite volume.

Anne Thomas: *The geometry of right-angled Coxeter groups.*

We will discuss several important concepts in geometric group theory, using right-angled Coxeter groups as examples. These concepts include quasi-isometry, hyperbolicity, nonpositive curvature, and the visual boundary. Right-angled Coxeter groups are a class of groups with particularly simple presentations, but as we show in joint work with Pallavi Dani, their geometry is surprisingly rich.

Lesley Ward: *Product Hardy spaces associated to operators with heat kernel bounds on spaces of homogeneous type.*

Much effort has been devoted to the generalisation of the Calderón–Zygmund theory in harmonic analysis from Euclidean spaces to metric measure spaces, or spaces of homogeneous type. Here the underlying space $\mathbb{R}^n$ with the Euclidean metric and Lebesgue measure is replaced by a set X equipped with a general metric, or quasi-metric, and a doubling measure. One aims to develop in this generality the full theory of function spaces and the operators that act on them. A further generalisation is to replace the Laplacian operator that underpins the Hardy spaces by more general operators L satisfying heat kernel estimates, such as Schrödinger operators, inverse square potentials, and second-order Maxwell operators with measurable coefficient matrices.

I will present some recent joint work with P. Chen, X.T. Duong, J. Li and L.X. Yan along these lines. We develop the theory of product Hardy spaces $H^p_{L_1,L_2}(X_1 \times X_2)$, for $1 \leq p < \infty$, defined on products of spaces of homogeneous type, and associated to operators L_1, L_2 satisfying Davis–Gaffney estimates. We define these product Hardy spaces via appropriate square functions. For $p = 1$, we define an atomic Hardy space and show that it coincides with the square function version. We establish a Calderón–Zygmund decomposition, and use it to prove interpolation results for these product Hardy spaces. We show that under the assumption of generalized Gaussian estimates, the product Hardy spaces coincide with the Lebesgue spaces, for an appropriate range of p.