Book of Abstracts
2012 International Workshop on Operator Theory and Applications

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Summary

Index of Speakers

Chapter 1

Plenary Talks

Speakers

1.1 Boundary value problems and Hardy spaces

Pascal Auscher
Université Paris 11 Orsay

Some recent works on the solvability of boundary value problems with L^2 data for elliptic systems use a first order formalism for a perturbed Dirac operator having a bounded holomorphic functional calculus. This has brought a new line of thoughts in this theory so far mainly developed for real equations through the notion of harmonic measure.

We shall review this first order approach and explain that in order to push this to L^p data for some p, this involves the use of the Hardy spaces associated with the perturbed Dirac operator.

1.2 Input/state/output linear systems and function theory: the Bergman space setting

Joseph A. Ball
Virginia Tech

The intertwining of discrete-time input/state/output (i/s/o) time-invariant linear systems and Hardy-space function theory on the unit disk $\mathbb{D}$ has been well known since the 1970s. A particular and central instance of this intertwining is the fact that Schur-class functions on the unit disk (i.e., holomorphic and contractive-operator valued functions on $\mathbb{D}$) arise as the transfer functions of a dissipative i/s/o linear system; the sub-class of inner functions (i.e., Schur-class with unitary boundary-value function on the unit circle) arise from conservative such linear systems whose state-update operator satisfies a stability condition.

Moreover, in the context of the Beurling-Lax-Halmos theorem associating an inner function Θ with a shift-invariant subspace $\mathcal{M}$ of the Hardy space H^2 so that $\mathcal{M} = \Theta H^2$, it is possible to construct the conservative linear system (A, B, C, D) having Θ as its transfer function explicitly in terms of operators constructed from the geometry of $\mathcal{M}$. These ideas arise in classical circuit theory, Sz.-Nagy–Foias operator-model theory and Lax-Phillips scattering theory.

Over the years there have been extensions of these ideas in many directions, but only recently has there been progress making systems-theory/function-theory connections in the context of Bergman-space inner functions in the work of Olofsson [1,2]. From the point of view of harmonic analysis, one can view part of the difficulty as the failure of the Plancherel theorem despite the ambient Hilbert-space 2-norm structure, i.e., the Z-transform (i.e., Fourier transform for the group Z) is no longer unitary.

In this talk we show how the classical Beurling theory for a shift-invariant subspace of the Hardy space has several distinct generalizations to the Bergman-space context, and how Bergman shift-invariant subspaces are parametrized in a canonical way by a certain class of *time-varying* conservative linear systems. This talk reports on on-going joint work with Vladimir Bolotnikov of the College of William and Mary (Williamsburg, VA, USA).

[1] A. Olofsson, A characteristic operator function for the class of n-hypercontractions, *J. Funct. Anal.* **236** (2006), 516–545.

[2] A. Olofsson, Operator-valued Bergman inner functions as transfer functions, *Algebra i Analiz.* **19** (2007) no. 4, 146–173.

1.3 Mexican hat completeness and invertibility of mixed frame operators

Qui Bui
University of Canterbury

The results presented in this talk are motivated by the following three observations/questions:

• For each $2 < p < \infty$, there exists a function $\theta \in \mathcal{S}(\mathbb{R})$ such that the affine system $A(\theta) = \{\theta_{j,k} = 2^{j/2}\theta(2^j\cdot)\}_{j,k\in\mathbb{Z}}$ forms a frame in $L^2(\mathbb{R})$ but is not complete in L^p (Lemarié-Tchamitchian).

• For each $1 < p < 2$, there exists a smooth and compactly supported function ψ such that $A(\psi)$ is a frame on $L^2(\mathbb{R})$ but the frame operator $f \mapsto \sum_{j,k} \langle f, \psi_{j,k}\rangle \psi_{j,k}$ is not invertible on $L^p(\mathbb{R})$ (Tao).

• For the Mexican hat function $\varphi(x) = (1 - x^2)e^{-x^2/2}$, is $A(\varphi)$ complete in $L^p(\mathbb{R})$ for any $p \neq 2$? (Meyer's completeness problem).

The first part of the talk will discuss the positive solution to the Meyer's problem. In the second part I'll present a criterion to show that, for ψ in a class of well-localized synthesizers, one can construct an analyzer ϕ such that the mixed frame operator $f \mapsto \sum_{j,k} \langle f, \phi_{j,k}\rangle \psi_{j,k}$ is invertible on $L^p(\mathbb{R})$ and on the Hardy space $H^1(\mathbb{R})$. This criterion is explicitly computable and is applied to prove the invertibility of the mixed frame operator associated with the Mexican hat function. The method in this part uses the Calderón-Zygmund theory and explicit bounds for the norms of certain singular integral operators.

Joint work with Richard Laugesen (University of Illinois, USA).

1.4 Recursively determined representing measures for bivariate truncated moment sequences

Raúl E. Curto
University of Iowa, USA

(This talk is based on joint work with Lawrence A. Fialkow)

For a degree $2d$ real bivariate moment sequence $\beta \equiv \beta^{(2d)} = \{\beta_{i,j}\}_{i,j\geq 0,\, i+j\leq 2d}$ to have a representing measure μ, it is necessary for the *associated moment matrix* M_d to be positive semidefinite and recursively generated. A theorem of Bayer and Teichmann implies that if β has a representing measure, then the associated moment matrix M_d admits positive, recursively generated moment matrix extensions $M_{d+1},\ M_{d+2}, \ldots$

For a bivariate recursively determinate moment matrix M_d, we show that the existence of positive, recursively generated extensions $M_{d+1}, \ldots, M_{2d-1}$ is sufficient for a measure. Examples illustrate that all of these extensions may be required to show that β has a measure.

We describe in detail a constructive procedure for determining whether such extensions exist. Under mild additional hypotheses, we show that M_d admits an extension M_{d+1} which has many of the properties of a positive, recursively generated extension.

1.5 Multiplication operators on Bergman space and Riemann surfaces

Ron Douglas
Texas A&M University

One fascination with the study of bounded operators on Hilbert spaces of holomorphic functions is the interactions with the underlying complex structure. If one considers the multiplication operator T_B by a finite Blaschke product $B(z)$ on the Hardy space, that is not apparent since the resulting operator is an isometry and the only invariant is the number n of zeros counted with multiplicity. In particular, the double commutant of T_z is isomorphic to the full matrix algebra $M_n(C)$.

On the other hand, for this same operator acting on the Bergman space on the unit disk, that is not the case. In joint works with Sun and Zheng and with Putinar and Wang, we show that the double commutant of M_z is isomorphic to C^q where q is the number of connected components of the Riemann surface $B(z) = B(w)$ contained in the bidisk.

In our talk we will discuss these examples and the larger significance of these results. Also, we will indicate the more complete details known in the case $B(z) = z^n$, where models can be exhibited for the restrictions of M_z to its minimal reducing subspaces. In the latter descriptions and the overall approach geometric ideas are critical.

1.6 Weighted BMO spaces associated to operators

Xuan Duong
Macquarie University

Let (X, d, μ) be a space of homogeneous type and $\{\mathcal{A}_t\}_{t>0}$, be a generalized approximations to the identity, for example $\{\mathcal{A}_t\}$ is a holomorphic semigroup e^{-tL} with Gaussian upper bounds generated by an operators L on $L^2(X)$.

In this talk, we introduce a weighted BMO space $\text{BMO}_{\mathcal{A}}(X, w)$ associated to the the family $\{\mathcal{A}_t\}$ and study the basic properties of this space including a weighted John-Nirenberg inequality and an interpolation theorem in scale of weighted L^p spaces. We also show that the dual space of the weighted Hardy space $H_L(X, w)$ associated to L (as in recent work of L. Song and L. Yan) is certain weighted BMO space $\text{BMO}_{L^*}(X, w)$ associated to the adjoint operator L^*.

As applications, we prove the boundedness of certain singular integrals with non-smooth kernels from the weighted L^∞ space to the weighted BMO space $\text{BMO}_L(X, w)$.

This is joint work with The Anh Bui.

1.7 A trace formula and stability of square root domains for non-self-adjoint operators

Fritz Gesztesy
University of Missouri, Columbia, USA

We extend the classical trace formula connecting the trace of resolvent differences of two (not necessarily self-adjoint) operators A and A_0 with the logarithmic derivative of the associated perturbation determinant from the standard case, where A and A_0 have comparable domains (i.e., one contains the other) to the case where their square root domains are comparable. This is done for a class of positive-type operators A, A_0.

We then prove an abstract result that permits to compare square root domains and apply this to the concrete case of 2nd order elliptic partial differential operators in divergence form on bounded Lipschitz domains.

This is based on various joint work with S. Hofmann, R. Nichols, and M. Zinchenko.

1.8 Free Banach spaces over metric spaces

Gilles Godefroy
Université Paris 6

The space of real-valued Lipschitz functions on a metric space M has a natural predual, namely the closed linear span of the Dirac measures. This space is usually called the free space over M. Its properties reflect quite accurately the features of the metric space M, and it allows a canonical linearization of the Lipschitz maps between metric spaces. Although their definition is simple, the structure of free spaces is not yet well-understood. We will show in particular the existence of compact metric spaces whose free space fails the approximation property, and investigate several related open questions.

1.9 On the structure of weighted shifts on directed trees

Il Bong Jung
Kyungpook National University, Korea

The main goal of this research is to implement some methods of graph theory into operator theory. We do it by introducing a new class of (not necessarily bounded) operators, which we propose to call weighted shifts on directed trees. We discuss some fundamental properties of such operators including closedness, adjoints, polar decomposition and moduli.

The relationships between domains of a weighted shift on a directed tree and its adjoint are described well. And formal normality, hyponormality, cohyponormality and subnormality of such operators are entirely characterized in terms of their weights and directed trees.

Moreover, a new method of verifying the subnormality of unbounded Hilbert space operators based on an approximation technique is proposed. In particular, we consider directed trees with one branching vertex and establish a connection of the subnormality with k-step backward extendability of bounded subnormal weighted shifts.

In addition, some interesting examples of weighted shifts on directed trees related to Stieltjes moment sequences are discussed. (This is a joint work with Z. Jablonski and J. Stochel)

1.10 The band method from operator theory and Szegő-Kreĭn orthogonal matrix functions

Rien Kaashoek
VU University Amsterdam

The band method is an abstract algebraic scheme in operator theory developed in the late eighties and early nineties, starting with pioneering work of H. Dym and I. Gohberg. This method allows one to deal with matrix-valued versions of classical interpolation problems, such as those of Schur, Carathéodory-Toeplitz and Nehari, from one point of view.

In the present talk the inverse theorems for Szegő-Kreĭn matrix polynomials and Kreĭn orthogonal entire matrix functions will be put into the band method setting and will be derived as corollaries of an abstract inverse theorem. Some other results will be given to illustrate the scope of the abstract approach.

The talk has a survey character and is based on recent joint work [1] with L. Lerer (Technion, Haifa).

[1] M.A. Kaashoek and L. Lerer, The band method and inverse problems for orthogonal matrix functions of Szegő-Kreĭn type, *Indagationes Math.*, to appear.

1.11 Odd K-theory and its applications

Jerry Kaminker
UC Davis

The K-theory of a space, X, comes in two varieties, even and odd, i.e. $K^1(X)$ and $K^0(X)$. The odd case is in some ways more natural and often has more interesting applications and descriptions.

By representing $K^1(X)$ as families of unbounded self-adjoint Fredholm operators one is led to a geometric approach to the study of their characteristic classes. In particular, it is possible to relate the vanishing of the components of the Chern character to the multiplicity of the spectrum of the operators in the family and to the manner in which the eigenspaces vary. For low dimensional spaces, X, one gets a complete answer. Using this approach in general raises many interesting questions.

On the homology side, one may view elements of $K_1(X)$ as Toeplitz extensions determined by a projection, $P \in \mathcal{B}(\mathcal{H})$, which commutes with a representation of $C(X)$ up to compacts. Such projections arise when studying elliptic differential operators on manifolds with boundary– namely as the Calderon projections onto the boundary values of solutions. Given two such projections that differ by a compact, the $K_1(X)$ elements will be equal, but the projections can be distinguished by their essential codimension. One can view this as a higher order invariant of the K-homology class and we will discuss how it can be obtained in a topological way. This is motivated by the study of Weinstein's question about defining an index for a contact diffeomorphism, and also by the work of Bojarski, Weber, Wojciechowski and Booss.

This is joint work with Ron Douglas, and also with Xiang Tang for the K-homology part.

1.12 Complete Positivity meets Linear Matrix Inequalities; a tale of Free Analysis

Igor Klep
The University of Auckland, New Zealand

Linear matrix inequalities (LMIs) are common in many areas: control systems, combinatorial optimization, statistics, etc. They often have unknowns $x = (x_1, \cdots, x_n)$ which are scalars, but in many problems the unknowns enter naturally as matrices. Given linear matrix inequalities L_1 and L_2 it is natural to ask:

$$\text{does one dominate the other; does } L_1(x) \succeq 0 \text{ imply } L_2(x) \succeq 0? \quad \text{(Q)}$$

In this talk we describe a natural relaxation of an LMI, based on substituting matrices for the variables x_j. With this relaxation, the domination question (Q) has an elegant answer. Indeed, for this "matricial" relaxation, a positive answer to (Q) is equivalent to the existence of matrices V_j such that

$$L_2(x) = V_1^T L_1(x) V_1 + \cdots + V_\mu^T L_1(x) V_\mu. \tag{A}$$

An observation at the core of this is that the relaxed LMI domination problem is equivalent to a classical problem in operator algebras. Namely, the problem of determining if a linear map is *completely positive*.

Algebraic certificates for positivity, such as (A) for linear polynomials, are typically called Positivstellensätze. We shall also present a nonlinear certificate: a noncommutative polynomial p is positive semidefinite on the matricial LMI domain $L(X) \succeq 0$ if and only if it has a weighted sum of squares representation with optimal degree bounds. Namely,

$$p(x) = s(x)^T s(x) + \sum_j f_j(x)^T L(x) f_j(x), \tag{B}$$

where $s(x), f_j(x)$ are vectors of polynomials of degree no greater than $\deg(p)/2$. A main ingredient of the proof is an analysis of extensions of Hankel matrices.

This is based on joint works with Bill Helton and Scott McCullough.

1.13 Ritt operators and their square functions on commutative and noncommutative L^p-spaces

Christian Le Merdy
University of Besançon (France)

Let (Ω, μ) be a measure space, let $1 < p < \infty$ and let $T\colon L^p(\Omega) \to L^p(\Omega)$ be a bounded operator. We consider the associated square function

$$\|x\|_T = \Big\| \Big(\sum_{k=1}^{\infty} k \big| T^k(x) - T^{k-1}(x) \big|^2 \Big)^{\frac{1}{2}} \Big\|_{L^p}$$

defined for any $x \in L^p(\Omega)$. We are interested in operators satisfying a "square function estimate" $\|x\|_T \leq K \|x\|_{L^p}$.

This issue is related to square functions appearing in martingale theory and in harmonic analysis. We give a functional calculus characterization of Ritt operators T such that T and T^* have a 'square function estimate', as well as applications and illustrations. We also consider similar questions on Hilbert spaces and on noncommutative L^p-spaces.

1.14 Subnormality of block Toeplitz operators

Woo Young Lee
Seoul National University

(This talk is based on joint work with Raúl E. Curto and In Sung Hwang)

Let $\mathcal{H}$ denote a separable complex Hilbert space and $\mathcal{B}(\mathcal{H})$ denote the set of all bounded linear operators acting on $\mathcal{H}$.

An operator $T \in \mathcal{B}(\mathcal{H})$ is said to be *subnormal* if there exists a Hilbert space $\mathcal{K}$ containing $\mathcal{H}$ and a normal operator N on $\mathcal{K}$ such that $N\mathcal{H} \subseteq \mathcal{H}$ and $T = N|_{\mathcal{H}}$. Loosely speaking, a subnormal operator is one that has a normal extension. But it is difficult to determine whether or not such a normal extension exists. Hence the following question is interesting and challenging: *Which operators are subnormal?*

The class of Toeplitz operators is a nice candidate for this question because Toeplitz operators are well-behaved. In 1970, P.R. Halmos addressed the following problem, so-called the Halmos' Problem 5 in his lectures "Ten problems in Hilbert space": *Is every subnormal Toeplitz operator either normal or analytic?* (Here analytic Toeplitz operators are by definition one with symbols of bounded analytic functions and any analytic Toeplitz operator is easily seen to be subnormal.)

In 1984, Halmos' Problem 5 was answered in the negative by C. Cowen and J. Long. Directly connected with it is the following question: *Which subnormal Toeplitz operators are normal or analytic?* M.B. Abrahamse has shown that the answer to the Halmos' question is affirmative for Toeplitz operators with bounded type symbols: i.e., every subnormal Toeplitz operator with a bounded type symbol (i.e., a quotient of two bounded analytic functions) is normal or analytic. In this talk we consider the following question: *Which subnormal block Toeplitz operators are normal or analytic?*

1.15 Elliptic PDEs in rough media: boundary regularity, elliptic measure, singular integrals, and properties of eigenfunctions

Svitlana Mayboroda
University of Minnesota

Elliptic boundary value problems are well-understood in the case when the boundary, the data, and the coefficients exhibit smoothness. However, perfectly uniform smooth systems do not exist in nature, and every real object inadvertently possesses irregularities (a sharp edge of the boundary, an abrupt change of the medium, a defect of the construction).

The analysis of general non-smooth elliptic PDEs gives rise to numerous challenges: possible failure of maximal principle and positivity, breakdown of boundary regularity, lack of well-posedness in L^2, to mention just a few. Further progress builds on a blend of harmonic analysis, potential theory, operator theory and geometric measure theory techniques.

In this talk we are going to discuss some highlights of the history, conjectures, surprising paradoxes, major methods, and recent results such as the higher-order Wiener criterion and maximum principle for higher order PDEs, solvability of rough elliptic boundary problems and perturbation in L^p, development of the new theory of Hardy spaces and analysis of singular integrals beyond the realm of the Calderon-Zygmund theory, as well as an intriguing phenomenon of localization of eigenfunctions of elliptic operators.

1.16 The stochastic Weiss conjecture

Jan van Neerven
TU Delft

The stochastic Weiss conjecture is an analogue, for linear stochastic evolution equation, of the celebrated Weiss conjecture in linear systems theory. It starting point is the observation that the stochastic Cauchy problem

$$dU(t) = AU(t)\,dt + B\,dW(t),$$

where A generates a strongly continuous semigroup $(S(t))_{t\geq 0}$ on some Banach space E, dW is a Gaussian white noise in a Hilbert space H, and B is a bounded operator from H into E, admits an invariant measure on E if and only if the operator-valued function $t \mapsto S(t)B$ enjoys a suitable Gaussian square summability property.

In analogy with the Weiss conjecture, one may then speculate that the latter should be characterised, under suitable additional assumptions, by an appropriate property of the mapping $\lambda^{1/2} \mapsto (\lambda - A)^{-1}B$.

In this talk, based on joint work with Jamil Abreu and Bernhard Haak, we will present a solution to the stochastic Weiss conjecture in case $-A$ admits a bounded H^∞-calculus of angle less than $\frac{\pi}{2}$ on E.

1.17 Singular Integral operators, commutators and weights: some recent progress

Carlos Perez
Universidad de Sevilla

It is well known that the basic operators from Harmonic Analysis are bounded on weighted Lp spaces when the weight satisfies the A_p condition. There is a lot of interest in understanding the behaviour of the operator norm in terms of the A_p constant of the weight.

In this expository lecture we plan to describe some recent progress on these and related questions for Singular Integrals operators like Calderón-Zygmund operators and their commutators with BMO functions.

1.18 Perturbed tuples of commuting self-adjoint operators

Vladimir Peller
Michigan State University

If f is a Borel function on $\mathbb{R}^n$ and $(A_1, A_2, \cdots, A_n)$ is an n-tuples of commuting self-adjoint operators, we apply f to this n-tuple and obtain the operator $f(A_1, A_2, \cdots, A_n)$. We study the behaviour of functions $f(A_1, A_2, \cdots, A_n)$ under perturbations of commuting n-tuples $(A_1, A_2, \cdots, A_n)$.

In particular, we find sharp conditions for f to be *operator Lipschitz*, i.e.,

$$\|f(A_1, A_2, \cdots, A_n) - f(B_1, B_2, \cdots, B_n)\| \\ \leq \text{const} \max_{1 \leq j \leq n} \|A_j - B_j\|,$$

where $(A_1, A_2, \cdots, A_n)$ and $(B_1, B_2, \cdots, B_n)$ are n-tuples of commuting self-adjoint operators.

I am going to discuss other related problems such as operator Hölder condition of order α, $0 < \alpha < 1$, perturbations in Schatten–von Neumann norms and other related problems.

Our results generalize earlier results by Peller, Aleksandrov–Peller, Aleksandrov–Peller–Potapov–Sukochev for functions of self-adjoint and normal operators. Note that the methods used in the case of self-adjoint and normal operators do not work in the case of n-tuples of commuting self-adjoint operators.

The lecture is based on my joint work with F. Nazarov.

1.19 Equilibrium states on operator-algebraic dynamical systems

Iain Raeburn
University of Otago

In operator-algebraic models in statistical mechanics, the equilibrium states are the states on the underlying operator algebra which satisfy the *KMS condition* of Kubo, Martin and Schwinger. There has recently been a great deal of interest in the computation of KMS states for dynamical systems arising in other areas of mathematics, and especially in number theory.

Here we will discuss this program, illustrating with a description of the KMS states on dynamical systems associated with self-coverings of tori.

This is joint work with Marcelo Laca and Jacqui Ramagge.

1.20 Asymptotics for Hessenberg matrices for the Bergman shift operator on Jordan regions

Edward B. Saff
Vanderbilt University

Let G be a bounded Jordan domain in the complex plane. The Bergman polynomials $\{p_n\}_{n=0}^{\infty}$ of G are the orthonormal polynomials with respect to the area measure over G. They are uniquely defined by the entries of an infinite upper Hessenberg matrix M. This matrix represents the Bergman shift operator of G. The main purpose of the paper is to describe and analyze a close relation between M and the Toeplitz matrix with symbol the normalized conformal map of the exterior of the unit circle onto the complement of $\overline{G}$. Our results are based on the strong asymptotics of p_n. As an application, we describe and analyze an algorithm for recovering the shape of G from its area moments.

1.21 Boundedness of bilinear multipliers and applications to bilinear Bochner-Riesz means

Lixin Yan
Sun Yat-Sen University

In this talk we will describe some recent progress on bilinear multipliers on R^n. We can weaken the regularity assumption for bilinear multipliers to assure the boundedness. As an application, we obtain some estimates on the bilinear Bochner-Riesz means. This is a joint work with F. Bernicot, L. Grafakos and L. Song.

Chapter 2

Special Session Talks

2.1 Special session on Continuous and Discrete Clifford analysis

Speakers

2.1.1 The Schur algorithm and de Branges spaces in the slice holomorphic case

Daniel Alpay
Ben-Gurion University of the Negev, Israel

Schur analysis originates with the works of Schur from 1917, and has numerous relationships with other fields of mathematics, and in particular the theory of linear systems and signal theory. We study Schur analysis in the quaternionic setting using the theory of slice hyperholomorphic functions. We discuss reproducing kernels, positive definite functions in this setting and we show how they can be obtained in our setting using the extension operator and the slice regular product, and define Schur multipliers, and find their co-isometric realization in terms of the associated de Branges-Rovnyak space. The aim of the talk is to explain how most, if not all, of the questions in classical Schur analysis have counterparts in the slice hyperholomorphic setting.

This is joint work with Fabrizio Colombo and Irene Sabadini.

2.1.2 The Gaussian and other kernels in discrete Clifford analysis

Swanhild Bernstein
Institute of Applied Analysis, Technische Universität Bergakademie Freiberg

Discrete Clifford analysis is a higher dimensional discrete function theory, based on skew Weyl relations. The skew Weyl relations are used to obtain the discrete Dirac operator as a “square root” of the 5 star Laplacian. Based on some duality of functions and distributions in discrete Clifford analysis we will discuss the Gaussian function and the Gaussian distribution which gives a distributional fundamental solution to the heat equation as well as to the parabolic Dirac operator.

This is joint work with F. Baaske of TU Bergakademie Freiberg, and H. de Ridder and F. Sommen, of Ghent University.

2.1.3 Lax pairs in Clifford analysis

Paula Cerejeiras
Universidade de Aveiro

Lax pairs are a well-established tool for the study of non-stationary nonlinear PDE's. Basically, a pair of linear operators acting on a certain Hilbert space is said to be a Lax pair for a given non-stationary nonlinear PDE if that PDE arises as a compatibility condition between the two operators. Since Lax pairs are closely linked to spectral decompositions they are not easily obtainable in the context of Dirac operators due to the non-commutativity of the underlying algebraic structure. In this talk we propose a construction of Lax pairs using the Dirac operator in the context of Clifford analysis. The main idea here is to substitute the classic approach by the so-called AKNS method. We hope to demonstrate that it is possible to obtain Lax pairs for linear differential operators with polynomially generalized Dirac operators we also give an example for a non-linear PDE.

2.1.4 On some notions of spectra for n-tuples of operators

Fabrizio Colombo
Politecnico di Milano, Italy

The S-spectrum has been introduced for the definition of the S-functional calculus that includes both the quaternionic functional calculus and a calculus for n-tuples of non necessarily commuting operators. In the case of n-tuples of commuting operators there are two different functional calculi that are based on the notion of F-spectrum. The notion of right spectrum for right linear quaternionic operators has been widely used in the literature, especially in the context of quaternionic quantum mechanics. Moreover, several results in linear algebra, like the spectral theorem for quaternionic matrices, involves the right spectrum. In this talk we discuss some relations among the various notions of spectra.

2.1.5 Monogenic Appell sequences for Jacobi polynomials

David Eelbode
University of Antwerp

Classically, Appell sequences in a complex variable $z = x + iy \in \mathbb{C}$ are defined as sets of holomorphic polynomials $\{P_k(z) : k \in \mathbb{Z}^+\}$ satisfying the relation $P_k'(z) := kP_{k-1}(z)$, where each polynomial $P_k(z)$ is homogeneous of degree k and $P_0 \neq 0$. Denoting the derivation (or lowering) operator by means of $P = \partial_z$, one can also interpret these Appell sequences as representations for the Heisenberg-Weyl algebra $\mathfrak{h}_1$, provided there exists an associated raising operator M satisfying $MP_k(z) = P_{k+1}(z)$ for all $k \in \mathbb{Z}^+$.

Recently, the problem of constructing analogues of complex Appell sequences in harmonic (or Clifford) analysis has gained new interest. These sequences are defined as polynomial sets $\mathbb{V} = \text{span}(P_k(\underline{x}))_{k\geq 0}$ containing scalar-valued (resp. Clifford algebra-valued) harmonic (resp. monogenic) polynomials (i.e. null solutions for the Laplace or Dirac operator) in the vector variable $\underline{x}$, for which the lowering operator P is a differential operator belonging to the Clifford-Weyl algebra $\mathcal{W}_m^C = \text{Alg}\big(x_1, \cdots, x_m; \partial_{x_1}, \cdots, \partial_{x_m}\big) \otimes \mathbb{C}_m$. It turns out that some of these harmonic (monogenic) Appell sequences can be related to the branching problem for certain irreducible representations for the spin group, the construction of orthonormal bases for these spaces and generalizations of the classical Fueter theorem, etc.

In this lecture, we will introduce an abstract technique to generate Appell sequences starting from Verma modules for the Lie algebra $\mathfrak{sl}(2)$, and then apply these results to some recent examples in Clifford analysis (such as Gegenbauer and Jacobi polynomials). The latter example is also part of a collaboration with I. Caçao.

This is partially joint work with I. Caçao, University of Aveiro, Portugal.

2.1.6 Some problems in function theory arising from the Weyl functional calculus

Brian Jefferies
University of NSW

In 1990, J. Ryan showed how complex monogenic functions defined in $\mathbf{C}^{n+1}$ are associated with monogenic functions defined in $\mathbf{R}^{n+1}$ using the Clifford version of the Cauchy integral formula. Locally, one can use a Taylor series expansion about a point in $\mathbf{R}^{n+1}$ to establish the extension to some domain in $\mathbf{C}^{n+1}$.

The connection between holomorphic functions of several variables and monogenic functions is exactly what is needed for the construction of functional calculi of a finite system of operators, such as the n-tuple of differentiation operators on a Lipschitz surface in $\mathbf{R}^{n+1}$. The talk mentions a few examples and some unresolved issues arising from this connection.

2.1.7 Clifford Analysis on discrete tori

Uwe Kähler
University of Aveiro, Portugal

In recent years one can observe an increasing interest in obtaining discrete counterparts for various continuous structures, especially a discrete equivalent to continuous function theory. This is not only driven by the idea of creating numerical algorithms for different continuous methods of studying partial differential equations, but also for true discrete purposes, as can be seen, among others, by recent results of S. Smirnov in connecting complex discrete function theory with problems in probability and statistical physics. While such ideas are very much developed in the complex case the higher-dimensional case is yet underdeveloped. This is mainly due to the fact that while discrete complex analysis is under (more or less) continuous development since the 1940's higher-dimensional discrete analysis started effectively only in the eighties and nineties with the construction of discrete Dirac operators either for numerical methods of partial differential equations or for problems in physics.

The development of Discrete Clifford analysis as being a discrete counterpart to classic Clifford analysis only started quite recently. Hereby the main obstacle is that to create a Weyl-Heisenberg structure is not so easy in the discrete case due to the fact of needing two different difference operators to factorize the Star-Laplacian. Here we will show that this problem can be simplified in case of discrete tori, i.e. periodic lattices, where one has a direct connection with phase-space analysis over Galois fields.

2.1.8 A Hodge-Dirac operator and its stable discretization

Paul C. Leopardi
Australian National University

Recent work by Arnold, Falk and Winther, and others, has resulted in a theory and family of methods called Finite Element Exterior Calculus. This theory treats problems, such as the mixed formulation of the Poisson problem, in an abstract framework involving differential forms, the exterior derivative d, Hodge decomposition and Hilbert complexes.

Since the Hodge-Dirac operator can be expressed as $\mathrm{d} + \mathrm{d}^*$, where d^* is the adjoint of d in a suitable space, an abstract variational problem involving the Hodge-Dirac operator can be addressed using the theory and methods of Finite Element Exterior Calculus.

This is joint work with Ari Stern.

2.1.9 Riesz Transforms and Steerable Wavelet Frames

Peter Massopust
Technische Universität München

The one-dimensional concept of an analytical wavelet is extended to higher dimensions. This extension, called a monogenic wavelet, is compatible with the rotation group. The construction of such monogenic wavelets is based on hypercomplex monogenic signals that are defined using higher-dimensional Riesz transforms. Employing methods from the theory of Clifford algebras, it is shown that this new class of monogenic wavelets generates steerable wavelet frames. Applications to descreening and contrast enhancement are presented.

2.1.10 The fundamental solution of higher spin Dirac operators

Tim Raeymaekers
Ghent University

In this talk, we will define the higher spin Dirac operators $\mathcal{Q}_\lambda$, with $\lambda = (l_1, \ldots, l_p)$ a highest weight for an irreducible Spin-representation. These operators act on a function $f(x)$ on $\mathbb{R}^m$, which takes values in more complicated representations $\mathcal{S}_\lambda$ of the spin group. They should be seen as generalizations of the classical Dirac operator and the Rarita-Schwinger operator. We shall construct the fundamental solution $e_\lambda(x)$ of these operators, satisfying the equation $\mathcal{Q}_\lambda e_\lambda(x) P_\lambda = \delta(x) P_\lambda$, using distribution theory and techniques coming from representation theory, for each $P_\lambda \in \mathcal{S}_\lambda$, and $\delta(x)$ being the Dirac delta distribution. In view of proving the classical integral formulae with this fundamental solution, we choose P_λ to be the reproducing kernel, defined by using the proper inner product.

This is joint work with David Eelbode.

2.1.11 Bergman spaces of slice hyperholomorphic functions

Irene Sabadini
Politecnico di Milano

Bergman theory admits two possible settings for the class of slice hyperholomorphic functions. In the so-called Bergman theory of the first kind, we provide a Bergman kernel which is defined on Ω (which is a subset of $\mathbb{H}$ if we are considering slice regular functions of a quaternionic variable or it is a subset of the Euclidean space $\mathbb{R}^{n+1}$ in case we consider slice monogenic functions with values in a Clifford algebra) and is a reproducing kernel. We discuss some properties of this theory. In the so-called slice hyperholomorphic Bergman theory of the second kind, we use the Representation Formula to define another Bergman kernel; this time the kernel is still defined on Ω but it is a reproducing kernel via an integral computed on the intersection of Ω with suitable complex planes and the integral does not depend on the choice of the chosen complex plane. In this framework it is possible to write in closed form the Bergman kernel on the unit ball and on the half space defined by the real positive numbers.

2.2 Special session on Toeplitz Operators and their Applications

Speakers

2.2.1

Miroslav Englis
Academy of Sciences, Czech Republic

2.2.2 Normal block Toeplitz operators

Dong-O Kang
Seoul National University

In 1998, Caixing Gu and Dechao Zheng characterized normality of block Toeplitz operators, where, the characterization was not so simple nor so easy to verify as that of scalar normal Toeplitz operators by A. Brown and P. Halmos. After several years, they gave a simple characterization of normality of a block Toeplitz operator T_Φ when the determinant of the coanalytic part of Φ is a non-zero function. In this note, we will show that the same characterization is still valid for block Toeplitz operators with general matrix-valued symbols.

This talk is based on the joint work with In Hyoun Kim at Incheon National University.

2.2.3 On the algebras generated by Toeplitz operators with piecewise continuous symbols

Nikolai Vasilevski
CINVESTAV, Mexico City

We explain an apparent disagreement between the fact that the Fredholm symbol algebras of two different C^*-algebras generated by Toeplitz operators with piecewise continuous symbols, acting on the Hardy space and acting on the Bergman space, have the same Fredholm symbol algebras and the same symbol homomorphism on generating operator, and the fact that the initial generators of these algebras are not unitary equivalent modulo compact operators.

2.3 Special session on Complex geometry and Operator theory

Speakers

2.3.1 Realization and interpolation theory for the Herglotz-Agler class over the poly-right-halfplane

Joseph A. Ball
Department of Mathematics, Virginia Tech

The Schur-Agler class of functions is defined as the class of holomorphic functions S on the polydisk $\mathbb{D}^d$ for which

$$S(T_1, \dots, T_d)$$

has norm at most 1 whenever $T_1, \dots, T_d$ is a commutative tuple of strict contraction operators on a Hilbert space $\mathcal{H}$. The Herglotz-Agler class is the class of holomorphic functions H on the d-variable poly-right-halfplane for which $H(X_1, \ldots, X_d)$ has positive real part whenever $X_1, \ldots, X_d$ is a commutative family of operators with each having strictly positive real part. While the Herglotz-Agler class is just a linear-fractional transform of the Schur-Agler class and the realization theory for the Schur-Agler class (i.e., realization of S as the transfer function of a multidimensional conservative input/state/output linear system) is well understood, the realization theory for the Herglotz-Agler class is considerably more subtle, especially in the several-variable case. We discuss several approaches to the realization theory for the Herglotz-Agler class with special attention to the rational case, and also indicate connections with a homogeneous subclass (the so-called Bessmertnyi class) of the Herglotz-Agler class.

2.3.2 On sectorial classes of inverse Stieltjes functions

Sergey Belyi
Troy University

We introduce sectorial classes of inverse Stieltjes functions acting on finite-dimensional Hilbert space as well as scalar classes of inverse Stieltjes functions characterized by their limit values. It is shown that a function from these classes can be realized as the impedance function of an L-system whose associated operator $\tilde{\mathbb{A}}$ is sectorial. Moreover, it is established that the knowledge of the limit values of the scalar impedance function allows us to find an angle of sectoriality of operator $\tilde{\mathbb{A}}$ as well as the exact angle of sectoriality of the accretive main operator T of such a system. The corresponding new formulas connecting the limit values of the impedance function and the angle of sectoriality of $\tilde{\mathbb{A}}$ are provided. These results are illustrated by examples of the realizing L-systems based upon the Schrödinger operator on half-line.

The talk is based on a recent joint work with E. Tsekanovskiĭ (see [1]).

[1] S.V. Belyi, E.R. Tsekanovskiĭ. Sectorial classes of inverse Stieltjes functions and L-systems. *Methods of Functional Analysis and Topology*, Vol. 18, no. 3 (2012) (to appear).

2.3.3 The role of the fundamental operator in functional model and complete unitary invariance of a pure Γ-contraction

Tirthankar Bhattacharyya
Department of Mathematics, Indian Institute of Science

A pair of commuting operators (S, P) defined on a Hilbert space $\mathcal{H}$ for which the closed symmetrized bidisc

$$\Gamma = \{(z_1 + z_2, z_1 z_2) : |z_1| \leq 1, \, |z_2| \leq 1\} \subseteq \mathbb{C}^2,$$

is a spectral set is called a Γ-contraction in the literature. A Γ-contraction (S, P) is said to be pure if P is a pure contraction, i.e, $P^{*n} \to 0$ strongly as $n \to \infty$. Here we construct a functional model and produce a complete unitary invariant for a pure Γ-contraction. The key ingredient in these constructions is an operator, which is the unique solution of the operator equation

$$S - S^* P = D_P X D_P, \text{ where } X \in \mathcal{B}(\mathcal{D}_P),$$

and is called the fundamental operator of the Γ-contraction (S, P). This talk is based on joint work with Sourav Pal.

2.3.4 Extremal holomorphic maps and the symmetrised bidisc

Zinaida A. Lykova
School of Mathematics, Newcastle University

We introduce the class of n-extremal holomorphic maps, a class that generalises both finite Blaschke products and complex geodesics, and apply the notion to the finite interpolation problem for analytic functions from the open unit disc into the symmetrised bidisc Γ. We show that a well-known necessary condition for the solvability of such an interpolation problem is not sufficient whenever the number of interpolation nodes is 3 or greater. We introduce a sequence $\mathcal{C}_\nu, \nu \geq 0,$ of necessary conditions for solvability, prove that they are of strictly increasing strength and show that $\mathcal{C}_{n-3}$ is insufficient for the solvability of an n-point problem for $n \geq 3$.

We introduce a classification of rational Γ-inner functions, that is, analytic functions from the disc into Γ whose radial limits at almost all points on the unit circle lie in the distinguished boundary of Γ. The classes are related to n-extremality and the conditions $\mathcal{C}_\nu$; we present numerous strict inclusions between the classes. The talk is based on a joint work with Jim Agler and N. J. Young.

2.3.5 Homogeneous operators in the Cowen-Douglas class of the ball

Gadadhar Misra
Department of Mathematics, Indian Institute of Science

We describe, via the jet construction, a new class of commuting tuple $\mathbf{T}$ of operators in the Cowen-Douglas class which are homogeneous with respect to the automorphism group of the ball, that is, $\varphi \cdot \mathbf{T}$ is unitarily equivalent to $\mathbf{T}$ for all φ in the automorphism group of the ball, $\varphi \cdot \mathbf{T}$ is the natural action of φ on $\mathbf{T}$ via the holomorphic functional calculus.

2.3.6 On Gamma contractions

Sourav Pal
Stat-Math Unit, Indian Statistical Institute

A pair of bounded operators (S, P) for which the closed symmetrized bidisc

$$\Gamma = \{(z_1 + z_2, z_1 z_2) : |z_1| \leq 1, |z_2| \leq 1\} \subseteq \mathbb{C}^2$$

is a spectral set, is called a Γ-contraction in the literature. For a contraction P and a bounded commutant S of P, let us consider the operator equation

$$S - S^* P = (I - P^* P)^{\frac{1}{2}} X (I - P^* P)^{\frac{1}{2}},$$

where X is a bounded operator on $\overline{\text{Ran}}(I - P^* P)^{\frac{1}{2}}$ with numerical radius of X being not greater than 1. We show that the existence and uniqueness of solution to the operator equation for a Γ-contraction (S, P) leads us to an explicit construction of a minimal Γ-isometric dilation of the

Γ-contraction (S, P). Hence it follows that the existence and uniqueness of solution to the above operator equation is a necessary and sufficient condition for the set Γ to be a spectral set for a pair of bounded operators.

2.3.7 Quotient Modules in the Cowen-Douglas class

Jaydeb Sarkar
Stat-Math Unit, Indian Statistical Institute

For a contractive multiplication operator on a space of holomorphic functions on the unit disk, there are two distinct approaches to models and their associated invariants, one due to Sz.-Nagy and Foias and the other due to Cowen and Douglas. This talk will discuss a "relation" between the sets of invariants obtained in these approaches in several variables context. In particular, our framework will allow the Hardy space, the Bergman and weighted Bergman spaces over the ball or polydisk and the Drury-Arveson space as building blocks of the quotient Hilbert modules. Also we will discuss the issues of similarity and unitary equivalence for a class of quotient Hilbert modules. This talk will be based mainly on joint work with R Douglas, Yun-Su Kim and H. Kwon.

2.3.8 Operator algebraic complex geometry

Orr Shalit
Department of Mathematics, Ben-Gurion University of the Negev

Let M be an algebra of holomorphic functions on the unit ball in complex n-space. Let V be an analytic variety in the ball. Restricting the algebra M to V, one gets an algebra of functions on V, call it $M(V)$.

Problem: how does the geometry of V determine the structure of $M(V)$?

In general, it depends what one means by "geometry" and what one means by "structure". We are interested in the operator algebraic, the Banach algebraic and the sheer algebraic structures of $M(V)$, and these different types of structure correspond to different interpretations of geometry. In the special case where M is the algebra of multipliers on Drury-Arveson space, and V is a homogeneous algebraic variety, we can say that the geometry of V completely determines the structure of $M(V)$, and vice-versa. When V is not homogeneous the situation is far more delicate, as I will explain. Our investigations on this matter have also led us to results in complex geometry of independent interest.

The talk is based on joint work with Ken Davidson and Chris Ramsey.

2.3.9 Domains associated with mu-synthesis and magic functions

Nicholas Young
Department of Pure Mathematics, Leeds University and School of Mathematics and Statistics, Newcastle University

The problem of mu-synthesis arises in H^∞ control. It is a species of interpolation problem which generalizes classical problems, such as those of Nevanlinna-Pick and Carathéodory-Fejér, but is much harder. Attempts to solve this problem have led to the study of some previously unfamiliar domains in $\mathbb{C}^n$. Two of these, the symmetrised polydisc and the tetrablock, have now been extensively studied by both operator-theorists and specialists in several complex variables, but there are others too. I shall describe some of the background to these domains, give some examples, describe the notion of magic functions of a domain and explain its connection with complex geometry, and particularly with the problem of whether the invariant distances on a given domain coincide.

2.4 Special session on Systems and Control Theory

Speakers

2.4.1 Linear Matrix Inequalities in Matrix Variables

Bill Helton
University of California San Diego

Bill's talks at IWOTA will describe the recent development of free analogs of two different classical subjects, real algebraic geometry and convex optimization. This one focuses on recent developments surrounding LMIs which have unknowns which are matrices. It will be co-ordinated with Igor Klep's plenary talk.

One of the main developments in optimization in the last 15 years is the rise of Linear Matrix Inequalities (LMIs). This tool now extends into most areas of science and engineering. Any problem treatable by LMIs is convex, but what about the converse? Convexity of a *free semi-algebraic set* $D := \{X : p(X) \succ 0\}$ is of serious interest. Convexity turns out to be a very strong hypothesis. For instance, if D is convex, bounded and contains 0, then D is the set of solutions to some (monic) LMI. Thus for free situations convexity and LMI techniques have the same scope. Less well understood are the possibilities for mappings between convex and not necessarily convex semi-algebraic sets as well as construction of free convex hulls.

A bit will be said about motivation in optimization and engineering. System engineering problems described by a signal flow diagram lead to a feasible set described by free inequalities. In this setting, convexity is highly desirable, because it guarantees reliability in a numerical optimization. In another direction, free considerations provide a systematic framework for "relaxing" basic problems regarding LMIs. For example, these encompass Nemirovski's relaxation of the matrix cube problem arising in spectral synthesis. While relating to a variety of areas, the ideas and techniques are functional analytic in nature and have a decided operator systems - operator spaces and matrix convexity flavor.

The talk describes recent results obtained jointly with Igor Klep, Scott McCullough, Harry Dym, Damon Hay, Chris Nelson, Nick Slingeland and Victor Vinnikov. For references see the arXiv.

2.4.2 Towards a Quantum-Classical Systems Theory

Matthew James
Australian National University

Classical systems and control theory is well developed with strong intellectual foundations and wide ranging applicability in engineering and elsewhere. Quantum technology is rapidly developing in laboratories around the world and quantum control theory has been progressing correspondingly. However, the approaches to quantum mechanics commonly used in physics differ significantly from classical systems. Furthermore, quantum technologies inevitably involve some combination of classical and quantum devices. It is therefore desirable to have an expanded framework for systems and control theory that seamlessly includes both quantum and classical systems. This talk will discuss these ideas using the framework of quantum probability.

2.4.3 Linear Programming and Averaging Approaches to Singularly Perturbed Optimal Control Problems

Vladimir Gaitsgory
University of South Australia

It has been some time since it was understood that a reduction technique based on equating of the singular perturbations parameter to zero may not lead to "near optimality" in nonlinear singularly perturbed (SP) optimal control problems (OCPs) and that, if this is the case, then an appropriate way of dealing with such problems is an averaging approach.

In this presentation, we will revisit some of the existing results and discuss new results on averaging of SP OCPs based on linear programming relaxations of the latter. Theoretical results will be illustrated with numerical examples.

2.4.4 Robust Stability of Quantum Systems with a Nonlinear Coupling Operator

Ian Petersen
The University of New South Wales at ADFA

This talk considers the problem of robust stability for a class of uncertain quantum systems subject to unknown perturbations in the system coupling operator. A general stability result is given for a class of perturbations to the system coupling operator. Then, the special case of a nominal linear quantum system is considered with non-linear perturbations to the system coupling operator. In this case, a robust stability condition is given in terms of a scaled strict bounded real condition.

2.4.5 Averaging Analysis Off SO(3); Stability of an Adaptive Attitude Estimation Algorithm

Victor Solo
The University of New South Wales

The attitude or pose of a rigid body is its orientation with respect to a fixed reference frame. In applications such as computer vision, robotics and satellite tracking the attitude has to be estimated in real time from body centered position measurements as well as known reference measurements such as star sight measurements. Other observations may also be available such as angular velocity. The fundamental object of interest is a time varying rotation matrix describing the rigid body motion. Here we sketch an averaging stability analysis of a new algorithm which is unusual in not estimating the rotation matrix (or its quaternion equivalent) directly.

2.4.6 Asymptotic Completeness for Weak Markov Processes

Rolf Gohm
Aberystwyth University

A weak Markov process in quantum probability for a discrete time parameter is essentially the same thing as a dilation of a row contraction by a row isometry. But the quantum probabilistic point of view suggests interesting additional questions. In this presentation we define subprocesses and quotient processes and the notion of a γ-cascade which allows a classification of the resulting structures. Considering the control theoretic notion of observability is particularly interesting for such a cascade. We show that if we start from a (noncommutative) Markov chain with an invariant state then we automatically get an associated subprocess and the problem of observability in this case is closely related to a theory of asymptotic completeness for Markov chains. This motivates a general definition of asymptotic completeness in the category of weak processes. Reference: R.G., Weak Markov Processes as Linear Systems, preprint http://arxiv.org/abs/1206.0378

2.4.7 Eigencurves of non-definite Sturm-Liouville problems for the p-Laplacian

Bruce Watson
University of the Witwaterstrand

The (λ, μ) eigencurves are studied for an indefinite weight quasi-linear Sturm-Liouville-type problem of the form

$$-\Delta_p y = (p-1)(\lambda r - q - \mu s)\operatorname{sgn} y\,|y|^{p-1}, \quad \text{on} \;\; (0,1)$$

with Sturmian-type boundary conditions (Δ_p being the p-Laplacian). Joint work with Paul A. Binding and Patrick J. Browne.

2.4.8 Fractional Linear Transformation for Quantum Feedback Networks

John Gough
Aberystwyth University

We present the theory of quantum feedback networks, and focus on the algebraic features emerging from model reduction through adiabatic elimination of slow degrees of freedom, feedback reduction, etc.

2.4.9 Linear State Estimation Via Multiple Sensors Over Rate-Constrained Channels

Subhrakanti Dey
University of Melbourne

In this talk we will consider state estimation of an unstable system using multiple sensors, where the sensors quantize their individual innovations (based on their own Kalman filtered estimates), which are then combined at the fusion centre to form a global state estimate. We obtain an asymptotic expression for the error covariance (or mean squared error) that relates the system parameters and bit rates used by the different sensors. Numerical results show close agreement with the true mean squared error for quantization at even moderate rates. An optimal rate allocation problem amongst the different sensors will also be considered and results presented. This initial study is the first to establish a relationship between rate of quantization and estimation error for linear dynamical system in a multi-terminal setting. Various lines of further investigations will also be discussed.

2.4.10 On Structure Preserving Transformations of the Itō Generator Matrix for Model Reduction of Quantum Feedback Networks

Hendra Nurdin
The University of New South Wales

Two standard operations of model reduction for quantum feedback networks, elimination of internal connections under the instantaneous feedback limit, and adiabatic elimination of fast degrees of freedom, are cast as structure preserving transformations of Itō generator matrices. It is shown, under certain technical conditions, that the order in which they are applied is inconsequential. That is, the two model reduction operations can be commuted.

2.4.11 Quadratic eigenvalue problems for second order systems

Sonja Currie
University of the Witwaterstrand

We consider the spectral structure of a quadratic second order system boundary-value problem. In particular we show that all but a finite number of the eigenvalues are real and semi-simple. We develop the eigencurve theory for these problems and show that the order of contact between an eigencurve and the parabola gives the Jordan chain associated with the eigenvector corresponding to that eigencurve. Following this we use our knowledge of the eigencurves to obtain eigenvalue asymptotics. Finally the completeness of the eigenfunctions is studied using operator matrix techniques. It should be noted here that the usual left definiteness assumptions have been overcome in this study.

2.5 Special session on Topics in Noncommutative Analysis

Speakers

2.5.1 Diagonals of positive compact operators, Schur-Horn generalizations and infinite majorization theory

Gary Weiss
University of Cincinnati

A brief summary of a recent full characterization in terms of majorization and further results, problems and connections (Joint work with V. Kaftal).

[1] V. Kaftal, G. Weiss, "An infinite dimensional Schur-Horn theorem and majorization theory", *J. Funct. Anal.*, *259* (2010), no. 12, 3115–3162.

2.5.2 Singular traces in symmetrically normed operator ideals

Fedor Sukochev
University of New South Wales

A trace of a matrix is known from the basic course of linear algebra – it is a linear functional on the algebra $M_n(\mathbb{C})$ (the algebra of all $n \times n$ matrices) which is invariant with respect to conjugations. In particular, it is unitarily invariant.

In 1909, Fredholm introduced a notion of a positive functional Tr extending the classical matrix trace in the setting of compact operators on an infinite-dimensional Hilbert space H. In 1932, J. von Neumann showed that Tr is a unitarily-invariant functional on the so-called trace class $\mathcal{S}_1$ (also called the ideal of all nuclear operators in the algebra $B(H)$ of all bounded linear operators on H).

It was unknown for a long time whether there are any other traces. The first example of the trace Tr_ω was proposed by Dixmier in 1966. The trace Tr_ω vanished on the ideal $\mathcal{S}_1$. Dixmier also identified a symmetrically-normed ideal of compact operators on which the trace Tr_ω is well-defined (and continuous). Let us recall that a two-sided ideal $\mathcal{I}$ of the algebra $B(H)$ is called symmetrically normed if

1. $\mathcal{I}$ is equipped with a Banach norm $\|\cdot\|_{\mathcal{I}}$.

2. If $A \in \mathcal{I}$ and $B \in B(H)$, then

$$\|AB\|_{\mathcal{I}} \leq \|A\|_{\mathcal{I}}\|B\|, \quad \|BA\|_{\mathcal{I}} \leq \|A\|_{\mathcal{I}}\|B\|.$$

All nontrivial ideals of $B(H)$ consist of compact operators on H. The following question arises naturally.

Q: Which symmetrically normed ideals of the algebra $B(H)$ carry a non-trivial trace (that is, a positive unitarily-invariant functional)?

Our main result [1] is as follows.

Theorem: Let $\mathcal{I}$ be a symmetrically normed operator ideal. The following conditions are equivalent:

1. There exists an operator $A \in \mathcal{I}$ such that
$$\lim_{n\to\infty} \frac{1}{n} \|A^{\oplus n}\|_{\mathcal{I}} > 0.$$
2. There exists a trace on $\mathcal{I}$.

[1] F. Sukochev, D. Zanin, "Traces on symmetrically normed operator ideals", *Crelle's journal,* in press.

2.5.3 On the distinction between Dixmier and Connes-Dixmier measurability

Alexandr Usachev
University of New South Wales

In the present talk we show that the classes of Dixmier and Connes-Dixmier traces differ even on the classical Dixmier ideal. Moreover, we construct a Marcinkiewicz operator space and a positive operator from this space which is Connes-Dixmier measurable but which is not Dixmier measurable.

2.5.4 On single-operators in II_1–factors: commutators and Schur-Horn

Ken Dykema
Texas A&M University

We consider the analogues in II_1–factors of two classical results in matrix algebras: (a) every traceless matrix is a single commutator and (b) the Schur-Horn theorem. Some partial results are described and some natural questions are asked. (The results for (a) are joint work with Anna Skripka, and for (b) with Junsheng Fang, Don Hadwin and Roger Smith.)

2.5.5 Spectral shift function – recent development

Denis Potapov
University of New South Wales

The talk will discuss spectral shift function associated with contractions. Two results are achieved recently in this area: resolution of an open problem by F. Gesztesy, A. Pushnitski and B. Simon and evolution of this result for the higher order spectral shift function.

2.5.6 On spectral flow inside essential spectrum

Nurulla Azamov
Flinders University

Spectral flow is by now a classical notion introduced by Atiyah, Patodi and Singer in 1976, and since then many hundreds of papers were published on this subject. Spectral flow is defined for paths of self-adjoint operators with compact resolvent, but it can also be defined for paths of self-adjoint operators outside of the common essential spectrum of the operators.

In this talk I shall discuss a new (to the best of my knowledge) notion, called (total) resonance index, that can be considered as a generalization of spectral flow into essential spectrum. Outside of essential spectrum resonance index coincides with spectral flow and spectral shift function, but unlike spectral flow it is defined inside essential spectrum too, and unlike the spectral shift function, it takes integer values a.e. inside essential spectrum. Resonance index is related to analytic properties of the scattering matrix as a function of coupling constant. For a fixed value of the spectral parameter (energy), scattering matrix is a meromorphic function of the coupling constant which may have real poles called resonances. When the energy is slightly shifted outside of the real axis, the pole may split into several poles and all of them necessarily get off the real axis. Resonance index is defined as the difference of the numbers of the shifted poles in upper and lower coupling constant half-planes. I shall also discuss two interpretations of the resonance index, as singular spectral shift function and as difference of winding numbers of the total phase shift of the scattering matrix corresponding to two different ways to calculate the total phase shift.

2.5.7 Upper triangular Toeplitz matrices and applications in II_1 factors

Ken Dykema
Texas A&M University

We show that there is a universal constant K such that, given a real sequence λ of length n that sums to zero, there is an upper triangular Toeplitz matrix T such that the eigenvalues of $T+T^*$ are λ and the norm of T is bounded by K times the supremum norm of λ. We apply this to show that certain self–adjoint elements of II_1–factors are the real parts of quasinilpotent elements in the II_1–factors (Joint work with Junsheng Fang and Anna Skripka).

2.5.8 Classification of finite-dimensional operator systems

Martin Argerami
University of Regina

Operator Systems are a unital selfadjoint subspaces of C*-algebras, considered as a category where the morphisms are the completely positive maps. Here we study operator systems generated by a single operator. Such operator systems are necessarily 2 or 3 dimensional, a fact that would make one think that they are easy to understand: after all, all 3-dimensional complex vector spaces are isomorphic, and all 3-dimensional

C*-algebras are isomorphic. It turns out that no such easy characterization exists. We will address this problem, and the problem of finding the minimal C*-algebra generated by such an operator system.

2.5.9 Subideals of operators

Gary Weiss
University of Cincinnati

In memory of Mihaly Bakonyi.

The talk will discuss recent results extending [1]. Subideals are ideals inside $B(H)$-ideals, a subject initiated by the 1983 work of Fong and Radjavi on principal ideals inside $K(H)$. Here we characterize all principal, finitely generated and countably generated subideals, and assuming falsity of the continuum hypothesis, all subideals generated by less than c elements (Joint work with S. Patnaik).

[1] S. Patnaik, G. Weiss, "Subideals of Operators", *J. Operator Theory,* in press.

[2] S. Patnaik, G. Weiss, "Subideals of Operators, II", preprint.

2.6 Special session on Harmonic Analysis of Differential Operators

Speakers

2.6.1 Hardy's Uncertainty Principle

Michael Cowling
University of NSW

Hardy's theorem states that, given a function f on $\mathbb{R}$, if

$$f(x) = O(\exp(-x^2/2)) \quad \text{and} \quad \hat{f}(y) = O(\exp(-y^2/2)),$$

then

$$f(x) = C\exp(-x^2/2).$$

Hardy's proof uses complex analysis, which does not lend itself to generalisations, and for some years there has been a search for a "simpler" "real variable" or "functional analytic" proof. We survey progress on this problem.

2.6.2 The Schrödinger equation for the sublaplacian on complex spheres

Valentina Casarino
Università di Padova

In this talk, based on a joint work with Marco Peloso, we discuss the Schrödinger equation for the sublaplacian on the unit sphere S^{2n+1} in C^{n+1}. Our study hinges on Strichartz estimates with fractional loss of derivatives for the solution of the linear Schroedinger equation. An essential tool in the proof is given by dispersive estimates for data that are spectrally localized. Our results should be compared with the estimates proved by Burq, Gerard and Tzvetkov in the Riemannian framework (see [B]).

[B] N. Burq, P. Gérard and N. Tzvetkov, Strichartz inequalities and the nonlinear Schrödinger equation on compact manifold, *Amer. J. Math. 126* (2004), 569–605.

[CP] V. Casarino and M. M. Peloso, The Schrödinger equation on complex spheres, preprint.

2.6.3 The Kato Square Root Problem on Submanifolds

Andrew Morris
University of Missouri

We solve the Kato square root problem for divergence form operators on complete Riemannian manifolds that are embedded in Euclidean space with a bounded second fundamental form. We do this by proving local quadratic estimates for perturbations of certain first-order differential operators that act on the trivial bundle over a complete Riemannian manifold with at most exponential volume growth and on which a local Poincaré inequality holds. This is based on the framework for Dirac type operators that was introduced by Axelsson-Rosén, Keith and McIntosh.

2.6.4 The stochastic functional calculus in L^p-spaces

Brian Jefferies
University of NSW

For any bounded linear operators A, B on a Banach space, *Feynman's operational calculus* associates a bounded linear operator $f_{\mu_1,\mu_2}(A,B)$ with a holomorphic function f and continuous probability measures μ_1, μ_2 on the interval $[0,1]$. The *time-ordering* measures μ_1, μ_2 index different choices of the functional calculus $f \longmapsto f_{\mu_1,\mu_2}(A,B)$.

The idea is extended in two directions: when A and B are allowed to be unbounded linear operators and when μ_1 is a continuous measure on an interval $[0,T]$ and μ_2 is replaced by Brownian motion W on $[0,T]$. If A satisfies square function estimates in an L^p-space $(1 < p < \infty)$

and B is a small perturbation of A in the square function norm, then $f \longmapsto f_{\lambda,W}(A+B)$ is defined for H^∞-functions on a sector in $\mathbf{C}$. In particular, $X = \exp_{\lambda,W}(A+B)x_0$ is a solution of the stochastic evolution equation

$$dX_t = AX_t\,dt + BX_t\,dW_t, \quad X_0 = x_0.$$

2.6.5 A restriction theorem for Metivier group

Paolo Ciatti
Università di Padova

I will discuss a recent result, obtained in collaboration with V. Casarino, concerning the mapping properties between Lebesgue spaces of the operators arising in the spectral resolution of the sublaplacians on groups of Metivier type.

2.6.6 Restriction estimates, sharp spectral multipliers and Bochner-Riesz means

Peng Chen
Macquarie University

We consider abstract non-negative self-adjoint operators on $L^2(X)$ which satisfy the finite speed propagation property for the corresponding wave equation. For such operators a restriction type condition is introduced which in the case of the standard Laplace operator is equivalent to $(p,2)$ restriction estimate of Stein and Tomas. In the considered abstract settings, the restriction type condition implies sharp spectral multipliers and endpoint estimates for the Bochner-Riesz summability. Several examples are given such as Schrödinger operators with inverse square potentials on $\mathbb{R}^n$, the harmonic oscillator, elliptic operators on compact manifolds and Schrödinger operators on asymptotically conic manifolds.

2.6.7 Weighted estimates for second order Riesz transforms associated to the Schrödinger operator

Fu Ken Ly
Macquarie University

We study the second order Riesz transforms $\partial_j \partial_k L^{-1}$, $1 \leq j, k \leq n$, associated to the Schrödinger operator $L = -\Delta + V$ on $\mathbb{R}^n$, where V belongs to a reverse Hölder class RH_s for some $s > n/2$. We show that these operators are bounded on $L^p(w)$ where w belongs to a class of weights associated to L containing the A_p class recently introduced by Bongioanni, Harboure, and Salinas. This is achieved through some new heat kernel estimates and two-parameter good-λ inequalities for these weights.

2.6.8 A $T(1)$-Theorem for non-integral operators

Dorothee Frey
Karlsruhe Institute of Technology

In generalization of the famous $T(1)$-Theorem of David and Journé for Calderón-Zygmund operators, we present a characterization of L^2-boundedness of so called non-integral operators.

Let X be a space of homogeneous type and let L be a sectorial operator with bounded holomorphic functional calculus on $L^2(X)$. We assume that the semigroup $\{e^{-tL}\}_{t>0}$ satisfies Davies-Gaffney estimates. Associated to L are certain approximations of the identity. We call an operator T a non-integral operator if compositions involving T and these approximations satisfy certain weighted norm estimates. The Davies-Gaffney and the weighted norm estimates are together a substitute for the usual kernel estimates on T in Calderón-Zygmund theory.

Under the additional assumption that a vertical Littlewood-Paley-Stein square function associated to L is bounded on $L^2(X)$, we show that a non-integral operator T is bounded on $L^2(X)$ if and only if

$$T(1) \in BMO_L(X) \quad \text{and} \quad T^*(1) \in BMO_{L^*}(X).$$

Here, $BMO_L(X)$ and $BMO_{L^*}(X)$ denote the recently defined $BMO(X)$ spaces associated to L that generalize the space $BMO(X)$ of John and Nirenberg.

The main tool in the proof is a paraproduct constructed via holomorphic functional calculus. We give an application of the $T(1)$-Theorem to show $L^2(X)$-boundedness of a different type of paraproduct operators.

This is joint work with P. C. Kunstmann.

2.6.9 A classification theorem for Helfrich surfaces

James McCoy
University of Wollongong

We consider critical points of an energy functional which is the the sum of the Willmore energy, λ_1-weighted surface area, and λ_2-weighted volume, for surfaces immersed in $\mathbb{R}^3$. The energy coincides with the Helfrich functional with zero ‘spontaneous curvature’, a functional proposed for modelling the shape of the red blood cell. Our main result is a complete classification of all smooth immersed critical points of the functional with $\lambda_1 \geq 0$ and small L^2 norm of trace-free curvature. We also prove the non-existence of critical points of the functional for which the surface area and enclosed volume are positively weighted. This is joint work with Glen Wheeler.

2.6.10 Localization of eigenfunction expansions.

Christopher Meaney
Macquarie University

We survey some work by Brandolini and Colzani about localization of eigenfunction expansions for the Laplace-Beltrami operator on a compact Riemannian manifold. Their work followed on from earlier work by the speaker and Bastis in the case of compact rank one symmetric spaces.

2.7 General session

Speakers

2.7.1 Projections and approximation properties for nonseparable spaces

David Yost
University of Ballarat

Suppose a non-separable Banach space has a family of (bounded linear) operators with small ranges which approximate the identity. Need it have many (not necessarily small rank) projections? We attempt to present a uniform approach, for several possible definitions of "small".

This is joint work with Anatolij Plichko (Krakow University of Technology).

2.7.2 Characterizations of connections and means for positive operators

Pattrawut Chansangiam
Chulalongkorn University

An axiomatic theory of operator connections and operator means was given by Kubo and Ando in 1980. In this paper, we give various axiomatic characterizations of connections. Kubo-Ando means are characterized with respect to some usual properties of scalar means, their associated operator monotone functions and their associated Borel measures. We characterize symmetric connections with respect to representing measures. It follows that there is a one-to-one correspondence between symmetric connections and finite Borel measures which are invariant under inversion. So, each symmetric mean corresponds to a probability Borel measure that is invariant under inversion.

2.7.3 On the hyperreflexivity of power partial isometries

Marek Ptak
University of Agriculture in Krakow

The concept of reflexivity and hyperreflexivity arises from the problem of existence of a nontrivial invariant subspace for an operator on a Hilbert space. An operator is called reflexive if it has so many invariant subspaces that they determine the membership in the algebra generated by the given operator. An operator is hyperreflexive (much stronger property than reflexivity) if the usual distance from any operator to the algebra generated by the given operator can be controlled by the distance given by invariant subspaces.

A power partial isometry is an operator for which all its powers are partial isometries. Necessary and sufficient conditions for hyperreflexivity of completely non unitary power partial isometries are given.

Joint work with K. Piwowarczyk

2.7.4 Mixingales on Riesz space

Wen-Chi Kuo
University of the Witwatersrand

Mixingales are a generalisation of martingales and mixing sequences introduced by D.L. McLeish in L^2 spaces. The concept was extended to L^1 spaces by D.W. Andrews. Here we give a generalization to the measure free setting of Riesz spaces (vector lattices) and show that even in this general setting a weak law of large numbers can be proved.

This is joint work with Jessica Vardy and Bruce A Watson.

2.7.5 Stochastic processes in Riesz spaces: Jensen's and Doob's inequalities

J.J. Grobler
North-West University — Potchefstroom University

Let $\mathbb{F}$ be a conditional expectation defined on a Dedekind complete Riesz space $\mathfrak{E}$. We define for a real convex function defined on the spectral interval of an element $X \in \mathfrak{E}$ the element $g(X) \in \mathfrak{E}$ and we show that Jensen's inequality $\mathbb{F}(g(X)) \geq g(\mathbb{F}(X))$ holds. We use this to show that if $(X_k, \mathbb{F}_k)_{k=1}^N$ is a finite martingale, then, for every $\lambda > 0$ and $p > 1$ we have

$$\mathbb{F}_1(|X_N|^p) \leq \mathbb{F}_1[(\sup_k |X_k|)^p] \leq \left(\frac{p}{p-1}\right)^p \mathbb{F}_1(|X_N|^p).$$

2.7.6 Maps on essentially infinite idempotent operators

Lucijan Plevnik
University of Ljubljana

Let $\mathcal{H}$ be an infinite-dimensional real or complex Hilbert space and $\mathcal{I}_\infty(\mathcal{H})$ the set of all bounded linear idempotent operators on H with infinite-dimensional image and infinite-dimensional kernel. We characterize three types of maps on $\mathcal{I}_\infty(\mathcal{H})$, namely poset automorphisms, bijective maps preserving orthogonality in both directions, and bijective maps preserving commutativity in both directions.

2.7.7 Subnormality of unbounded composition operators in L^2-spaces

Piotr Budzynski
University of Agriculture in Krakow

We present a criterion for subnormality of unbounded composition operators acting in L^2-spaces. As we show, such an operator is subnormal whenever there exists a measurable family of probability measures satisfying the so-called consistency condition. It is the first criterion for subnormality of composition operators in the unbounded case. The consistency condition involves the Radon-Nikodym derivative of the first order only, as opposed to the well-known Lambert's characterization of subnormality of bounded composition operators, which means that our criterion is new even in the bounded case. We provide some examples and discuss few related results.

The talk is based on a joint work with Z. J. Jabłoński, I. B. Jung, and J. Stochel.

2.7.8 Quantitative Lyapunov theorem for abstract and concrete Banach spaces

Sergey Ajiev
University of New South Wales

In 1991, M. I. Kadets has quantified and extended one of Lyapunov's theorems on the range of measures by estimating the mid-point measure of non-convexity of the range of an atomic countably additive measure taking values in a B-convex Banach space. We strengthen this result in the setting of measures with values in various abstract and concrete Banach spaces obtaining explicit and occasionally sharp estimates for a stronger measure of non-convexity. The concrete spaces under consideration include various classes of anisotropic Besov, Lizorkin-Triebel and Sobolev spaces of functions defined on an open subset of an Euclidean space, a wide class of parameterised independently generated (IG) spaces, Schatten-von Neumann classes and more general parameterised non-commutative and non-commutative-valued spaces. The classical Lyapunov theorems found important applications, for example, in optimal control.

2.7.9 The form method for accretive operators

Manfred Sauter
University of Auckland

The term 'form method' describes a procedure in Hilbert space that allows to generate operators (or extensions of operators) with certain properties by a correspondence principle between appropriate sesquilinear forms and the desired operators. For example, the Riesz–Fréchet representation theorem provides an immediate one-to-one correspondence between continuous sesquilinear forms and bounded operators. The Lax–Milgram theorem then states that it is the invertible operators that are associated with the coercive, continuous sesquilinear forms.

For the generation of unbounded operators the assumption of ellipticity has proved fruitful. It was shown in the 1950s by Kato and

J.-L. Lions that there is a correspondence between m-sectorial operators in a Hilbert space H and elliptic forms with a form domain V that is a Hilbert space densely embedded in H. In the 1960s McIntosh generalised this by giving sufficient conditions for an accretive form to generate an m-accretive operator. Recently, Arendt and ter Elst showed that in the elliptic setting one can do without the requirement that the 'embedding' of V into H be injective.

In this talk we will showcase the pathological behaviour that can occur for general continuous accretive forms and non-injective 'embeddings', describe operators which can and cannot be generated and give conditions that make things more well-behaved.

This is joint work with Tom ter Elst and Hendrik Vogt.

2.7.10 Determination of a family of 1-dimensional Dirac systems from their spectrum

Thomas Roth
University of the Witwatersrand

The 1-dimensional Dirac equation with absolutely continuous potential is considered on the finite interval and reposed as a periodic problem on $\mathbb{R}$. Through analysis of the discriminant the interlacing structure for the eigenvalues of the problem with periodic, anti-periodic and Dirichlet boundary conditions is established. The following inverse problem is solved: If the eigenvalues of a Dirac boundary value problem are all double eigenvalues then its potential is constant.

This is joint work with Sonja Currie and Bruce A Watson.

2.7.11 On determinants associated with Hankel operators

Gordon Blower
University of Lancaster

Let $X : H \to K$ and $Y : K \to H$ be Hilbert-Schmidt operators. Then XY and YX are trace class, and they have Fredholm determinant $\det(I + XY) = \det(I + YX)$. Our aim in this talk is to use linear systems and operator determinants to solve nonlinear partial differential equations such as KdV

$$\frac{\partial u}{\partial t} = \frac{\partial^3 u}{\partial x^3} - 3u\frac{\partial u}{\partial x}.$$

For special meromorphic potentials u, the idea is to realise solutions of

$$-\frac{d^2\psi}{dx^2} + u\psi = \lambda\psi$$

in terms of linear systems $(-A; B; C)$. We introduce an algebraic structure on the linear systems which mirrors the algebraic properties of the potentials u. The method is effective in giving simple constructions of solutions and elliptic solutions in terms of block finite matrices.

2.7.12 Hardy-Littlewood inequalities for norms of positive operators on sequence spaces

Miguel Lacruz
Universidad de Sevilla

We consider estimates of Hardy and Littlewood for norms of operators on sequence spaces, and we apply a factorization result of Maurey to obtain improved estimates and simplified proofs for the special case of a positive operator.

2.7.13 Quotient Hilbert modules in the Cowen-Douglas class

Hyun Kwon
Seoul National University

We discuss some recent results obtained about quotient Hilbert modules over $\mathbb{C}[z_1, \cdots, z_n]$ that belong to the Cowen-Douglas class. Among them are the tensor product representation of the associated anti-holomorphic vector bundles and isomorphism and similarity characterizations. These descriptions are independent of the type of the basic Hilbert modules used in the construction of the quotient modules. This talk is based on joint work with R. G. Douglas, Y. Kim, and J. Sarkar.

2.8 Special session on Operators on spaces of Analytic Functions

Speakers

2.8.1 Truncated Toeplitz Operators

Isabelle Chalendar
Université Lyon I

According to Beurling's theorem, a coinvariant subspace of $H^2 = H^2(\mathbb{D})$ is a closed subspace of H^2 of the form $H^2 \cap (\Theta H^2)^\perp$, where Θ is an inner function, that is, a holomorphic and bounded function on the open unit disc D whose radial limits are of modulus one almost everywhere on the unit circle.

Compressions of Toeplitz operators to coinvariant subspaces of H^2 are called *truncated Toeplitz operators*. We study two questions related to these operators. The first, raised by Sarason, is whether boundedness of the operator implies the existence of a bounded symbol; the second is the Reproducing Kernel Thesis. We show that in general the answer to the first question is negative, and we exhibit some classes of spaces for which the answers to both questions are positive.

This is a joint work with A. Baranov, E. Fricain, J. Mashreghi and D. Timotin.

2.8.2 Rational dilation and complete sets of Pick kernels

Michael Dritschel
Newcastle University

It is a now classical result that the Szegő kernel is a complete Pick kernel for $H^\infty(\mathbb{D})$ (so that this kernel is the only one needed in formulating the Pick condition for interpolation with either scalar or matrix valued functions). It has also been long known that if the disk is a spectral set for a Hilbert space operator (so von Neumann's inequality holds for rational functions, or equivalently, the operator is contractive) then the operator dilates to a normal operator with spectrum on the boundary of the disk (a unitary operator). On the other hand, there are well known examples where both of these properties fail — for example, $H^\infty(\mathbb{D}^3)$. We look at some further examples, and explore the connection between complete sets of Pick kernels and rational dilation.

2.8.3 Nearly Invariant Subspaces of $L^2(\mathbb{R})$ and Symmetric Operators

Robert T. W. Martin
University of Capetown

A subspace S of the Hardy space of upper half-plane H^2 is said to be *nearly invariant for the backwards shift* if for any f in S that vanishes at i, $f(z)/(z-i)$ belongs to S. Any nearly invariant subspace S has the form $S = h(\Theta H^2)^\perp$ where Θ is an inner function vanishing at i, and h is an isometric multiplier of $(\Theta H^2)^\perp$ onto S. We call a subspace S' of L^2 of the real line nearly invariant if $S' = uS$ where u is a unimodular function and S' is a nearly invariant subspace of H^2.

In this talk we show that a subspace $S' = uhK^2_\Theta$ is nearly invariant with meromorphic inner function Θ if and only if the operator M of multiplication by the independent variable has a simple symmetric regular restriction to S with deficiency indices $(1,1)$. Such a subspace S is necessarily a reproducing kernel Hilbert space on $\mathbb{R}$ with a $\mathbb{T}$-parameter family of total orthogonal sets of point evaluation vectors. To achieve this result we will use the dilation theory of completely positive maps.

2.8.4 Spectrum of an Invertible Weighted Composition Operator with a Hyperbolic Composition Map

Gajath Gunatillake
American University of Sharjah

A weighted composition operator $C_{\psi,\varphi}$ takes an analytic map f on the open unit disc of the complex plane to the analytic map $\psi \cdot f \circ \varphi$

where φ is an analytic map of the open unit disc into itself and ψ is an analytic map on the open unit disc. In this talk we investigate the spectra of invertible weighted composition operators on H^2 whose composition map φ is a hyperbolic automorphism of the open unit disc.

2.8.5 Normal and co-hyponormal weighted composition operators on H^2

Sungeun Jung
Ewha Women's University, Seoul

In this talk, we consider normal and cohyponormal weighted composition operators on the Hardy space H^2. We show that if $W_{f,\varphi}$ is cohyponormal, then f is outer and φ is univalent. Moreover, we prove that when the composition map φ has an interior fixed point, $W_{f,\varphi}$ is cohyponormal if and only if it is normal; in this case, f and φ can be expressed as linear fractional maps. As a corollary, we find the polar decomposition of the cohyponormal operator $W_{f,\varphi}$. Finally, we examine the commutant of a cohyponormal weighted composition operator.

2.8.6 Numerical ranges of some block Toeplitz composition operators

Linda Patton
California Polytechnic University, San Luis Obispo

The composition operators on $H^2(\mathbb{D})$ with minimal polynomial $z^n - 1$ are shown to be block Toeplitz with Toeplitz symbol equal to an $n \times n$

matrix-valued polynomial of degree 1. This result is used to prove that the numerical range of a composition operator on $H^2(\mathbb{D})$ with minimal polynomial $z^3 - 1$ cannot be a circular disk.

2.8.7 Hilbert Matrix Operator on Spaces of Analytic Functions

Maria Nowak
Instytut Matematyki UMCS, Lublin

The Hilbert matrix is an infinite matrix H whose entries are $a_{n,k} = (n+k+1)^{-1}$. This matrix induces a linear operator on sequences:

$$H : (a_k)_{k \in N_0} \mapsto \left(\sum_{k=0}^{\infty} \frac{a_k}{n+k+1} \right)_{n \in N_0}$$

Apart from sequence spaces, the Hilbert matrix can be viewed as an operator on spaces of analytic functions by its action on their Taylor coefficients. If

$$f(z) = \sum_{k=0}^{\infty} \hat{f}(k) z^k$$

is a holomorphic function in the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$, then we define a transformation H by

$$Hf(z) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{\hat{f}(k)}{n+k+1} z^n$$

We consider the action of the Hilbert matrix operator, H, on the Hardy space H^1, weighted Hardy spaces H^p_α for $\alpha \geq 0$, Bergman spaces with logarithmic weights, etc. In particular, we extend the Diamantopulos-Siskakis result by proving that H maps H^p_α into H^p_α if and only if $\alpha + 1/p < 1$. A criterion for Hf to belong to H^1 is given, provided the coefficients of f are nonnegative. Also, H maps the A^2-space with weight $\log^\alpha(2/(1-|z|))$ into the ordinary Bergman space A^2 if $\alpha > 3$. Similarly, the Bloch space with logarithmic weight is mapped by H into the ordinary Bloch space.

2.8.8 Composition operators on weighted Bergman spaces with admissible Békollé weights

Sei-Ichiro Ueki
Ibaraki University, Hitachi

Let $\sigma(z)$ be radial, non-negative, non-increasing and continuous on the unit disk $\mathbb{D}$. For each $0 < p < \infty$, we consider the weighted Bergman space $A^p(\sigma\, dA)$ where dA denotes the normalized area measure on $\mathbb{D}$. When $\sigma(z)/(1-|z|^2)^\alpha$ $(\alpha > -1)$ satisfies the Békollé weight condition and $\sigma(r)/(1-r^2)^{1+\delta}$ is non-decreasing for some $\delta > 0$, we call this weight $\sigma(z)$ an admissible Békollé weight.

In this talk we will characterize the compactness of composition operators C_ϕ acting between weighted Bergman spaces with admissible Békollé weight in terms of a modified counting function.

2.8.9 Strongly Compact Algebras Associated with Composition Operators

Joel Shapiro
Portland State University

An algebra of bounded linear operators on a Hilbert space is called *strongly compact* whenever each of its bounded subsets is relatively compact in the strong operator topology. The concept, which has its roots in the search for invariant subspaces, is most commonly studied for the algebra generated by a single operator, and for the operator's commutant. In this talk the focus will be on the strong compactness of these two algebras when the operator in question is a composition operator induced on the Hardy space by a linear fractional self-map of the unit disc. In this setting, strong compactness will be completely characterized for the generated algebra, and "almost" characterized for the commutant, thus extending a recent investigation begun by Fern dez-Valles and Lacruz [A spectral condition for strong compactness, *J. Adv. Res. Pure Math.* 3 (4) 2011, 50–60]. Along the way it becomes necessary to consider strong compactness for algebras associated with multipliers, adjoint composition operators, and even the Cesàro operator.

2.8.10 Commutants of Analytic Multiplication Operators

Rebecca Wahl
Butler University, Indianapolis

If ψ is a bounded analytic function on the unit disk $\mathbb{D}$, the analytic multiplication operator, or Toeplitz operator, T_ψ, on a Hilbert space $\mathcal{H}$

of analytic functions on the disk is defined by $(T_\psi f)(z) = \psi(z)f(z)$. Several authors have studied the commutant of such operators on the Hardy Hilbert space, $H^2(\mathbb{D})$, but the topic is interesting in other spaces as well, such as the Bergman space $A^2(\mathbb{D})$.

In this talk we revisit some results and techniques used to determine the commutant of these operators on $H^2(\mathbb{D})$, and show that they may be adapted and used to determine the set of operators that commute with T_B when B is a finite Blaschke product. In particular, we show that the commutants of T_B acting on the Hardy space and T_B acting on the Bergman space are the same(!) for B a finite Blaschke product.

2.8.11 Commutators of Composition Operators with Adjoints of Composition Operators on Weighted Bergman Spaces

Rachel Weir
Allegheny College, Meadville

For linear-fractional self-maps φ and ψ of the unit disk $\mathbb{D}$, where at least one of φ and ψ is a non-automorphism, we show that the commutator $[C_\psi^*, C_\varphi]$ is non-trivially compact on the weighted Bergman space $A^2_\alpha(\mathbb{D})$ if and only if either φ and ψ are both parabolic or φ and ψ are both hyperbolic, with associated conclusions about their fixed points in each case. In the automorphism case, we show that this commutator is compact if and only if both φ and ψ are rotations.

This talk describes joint work with Barbara D. MacCluer and Sivaram K. Narayan.

2.8.12 Spectral Picture for Some Composition Operators

Carl C. Cowen
IUPUI, Indianapolis

If ϕ is an analytic map of the disk into itself and H is a Hilbert space of analytic functions on the disk, the composition operator C_ϕ is the operator given by $C_\phi f = f \circ \phi$ for f in H. In this talk, we will discuss the point spectrum of C_ϕ^* on H^2 when $\phi(0) = \phi'(0) = 0$ or more generally, when ϕ has a fixed point in the open disk, but ϕ is not locally univalent there. The (power-)compact case is easy:

$$\sigma(C_\phi^*) = \sigma_p(C_\phi^*) = \{0, 1\}$$

In her recent thesis, Maria Neophytou used work of Poggi-Corradini and the speaker to show that for a broad class of such composition operators, the point spectrum of C_ϕ^* is an open disk centered at the origin with radius $1/\sqrt{\phi'(b)}$ where b is a fixed point of ϕ on the unit circle such that

$$\phi'(b) = \min\{\phi'(c) : c \text{ is a fixed point of } \phi \text{ with } |c| = 1\}$$

Her proof will be outlined and a conjecture will be offered.

2.9 Special session on Operator Theory, Function Theory, and Linear systems

Speakers

2.9.1 Schur analysis in the case of slice-hyperholomorphic functions

Daniel Alpay
Ben-Gurion University of the Negev, Beer-Sheva, Israel:

We present work with Fabrizio Colombo and Irene Sabadini, see [1] and related preprints on Arxiv, where we develop Schur analysis, and in particular the Schur algorithm, and a theory of linear systems when the complex numbers are replaced by the skew-field of quaternions, using the recently developed theory of slice-hyperholomorphic functions [3]. There is a combination of a non-commutative setting (since the quaternions lack the commutativity property) and of analyticity (via the slice-hyperholomorphic functions). The novelty of our approach is that slice-hyperholomorphic functions allows to write realizations in terms of a suitable resolvent, the so called S-resolvent operator and to extend several results that hold in the complex case to the quaternionic case. We discuss reproducing kernels and positive definite functions in this setting. We define Schur multipliers, and find their co-isometric realization in terms of the associated de Branges-Rovnyak space. Differences with an earlier approach [2] which used series of Fueter polynomials will be explained.

[1] D. Alpay, F. Colombo and I. Sabadini. Schur functions and their realizations in the slice hyperholomorphic setting. Integral Equations and Operator Theory, vol. 72 (2012), pp. 253-289.

[2] D. Alpay, M. Shapiro and D. Volok. Rational hyperholomorphic functions in R4. Journal of Functional Analysis, vol. 221 (2005) pp. 122-149.

[3] G. Gentili and D. Struppa. A new theory of regular functions of a quaternionic variable. Adv. Math., 216(1):279–301, 2007.

2.9.2 Zero-pole interpolation, Beurling-Lax representations for shift-invariant subspaces, and transfer function realizations: half-plane/continuous time versions

Austin Amaya
Virginia Tech

Given a full-range simply-invariant shift-invariant subspace $\mathcal{M}$ of the vector-valued L^2 space $L^2_{\mathcal{U}}(\mathbb{T})$ over the unit circle, the classical Beurling-Lax-Halmos Theorem obtains a unitary operator-valued function W on $\mathbb{T}$ so that $\mathcal{M} = WH^2_{\mathcal{U}}$; in this case necessarily $\mathcal{M}^{\perp} = W\left(H^2_{\mathcal{U}}\right)^{\perp}$. The Beurling-Lax-Halmos Theorem of Ball-Helton [3] obtains such a representation for the case of a pair of shift-invariant subspaces $(\mathcal{M}, \mathcal{M}^{\times})$—with $\mathcal{M}$ forward full-range simply-invariant and $\mathcal{M}^{\times}$ backward full-range simply-invariant—forming a direct-sum decomposition of $L^2_{\mathcal{U}}(\mathbb{T})$ with a new almost everywhere invertible W on $\mathbb{T}$. For the case where $(\mathcal{M}, \mathcal{M}^{\times})$ is a finite-dimensional perturbation of the model pair $(H^2_{\mathcal{U}}(\mathbb{T}), H^2_{\mathcal{U}}(\mathbb{T})^{\perp})$, Ball-Cohen-Ran [1] (see also the book of Ball-Gohberg-Rodman [2]) obtained a transfer function realization formula for the representer W, parametrized from zero-pole data computed from $\mathcal{M}$ and $\mathcal{M}^{\times}$. Later work by Ball-Raney [4] extended this analysis to the nonrational case where the zero-pole data is taken in an appropriate infinite-dimensional operator-theoretic sense. Our current work obtains the analogue of these results for the case of a pair of subspaces $(\mathcal{M}, \mathcal{M}^{\times})$ of $L^2_{\mathcal{U}}(\mathbb{R})$ invariant under the forward and backward translation groups. These results rely on recent advances in the understanding of continuous-time infinite-dimensional input-state-output linear systems now codified in the book of Staffans [5].

[1] J.A. Ball, N. Cohen, and A.C.M.Ran, Inverse spectral problems for regular improper rational matrix functions, in *Topics in Interpolation Theory of Rational Matrix-Valued Functions* (ed. I.Gohberg), **OT 33** Birkhäuser-Verlag, Basel, 1988, pp.123–173.

[2] J.A. Ball, I.Gohberg, and L. Rodman, *Interpolation of Rational Matrix Functions*, **OT 45**, Birkhäuser-Verlag, Basel, 1990.

[3] J.A. Ball and J.W. Helton, Beurling-Lax representations using classical Lie groups with many applications, I. $GL(n, \mathbb{C})$ and Wiener-Hopf factorization, *Integral Equations and Operator Theory* **7** (1984), 291–203.

[4] J.A. Ball and M.W. Raney, Discrete-time dichotomous well-posed linear systems and generalized Schur-Nevanlinna-Pick interpolation, *Complex Analysis and Operator Theory* **1** (2007) no. 1, 1–54.

[5] O.Staffans, *Well-Posed Linear Systems*, Encyclopedia of Mathematics and Its Applications **103**, Cambridge University Press, 2005.

2.9.3 On invariant subspaces of the Hardy space of the polydisk

Tirthankar Bhattacharyya
Indian Institute of Science

Let $\mathbb{D}^d$ be the d-dimensional polydisk. Let $H^2(\mathbb{D}^d)$ be the Hardy space on it, i.e., the reproducing kernel Hilbert space with kernel

$$k(\underline{z},\underline{w}) = \frac{1}{(1 - z_1\overline{w}_1)(1 - z_2\overline{w}_2)\dots(1 - z_d\overline{w}_d)}.$$

Let M_{z_i} for $i = 1, 2, \dots, d$ denote the multiplication operators on $H^2(\mathbb{D}^d)$ by the co-ordinate functions. In this talk, we discuss the often studied question of describing common invariant subspaces of the M_{z_i}. Let $\mathcal{M}$ denote such an invariant subspace. The success stories so far are as follows.

1. Ahern and Clark handled the case when $\mathcal{M}$ has finite co-dimension.
2. Mandrekar showed that $\mathcal{M}$ is the range of an inner function if and only if the $M_{z_i}|_{\mathcal{M}}$ are doubly commuting.
3. Izuchi, Nakazi and Seto gave a complete description of what $\mathcal{M}$ looks like in case the compression of M_{z_i} to the orthocomplement of $\mathcal{M}$ are doubly commuting, but only for $d = 2$.

We study the last case, i.e., when the compression of M_{z_i} to the orthocomplement of $\mathcal{M}$ are doubly commuting for any dimension and also for any multiplicity. This means that our space is not just $H^2(\mathbb{D}^d)$, but $H^2(\mathbb{D}^d) \otimes E$ for some E. We list out all the possibilities for $\mathcal{M}$. This work is joint with E. K. Narayanan and J. Sarkar.

2.9.4 Two parameter methods for symmetrizable non-self-adjoint eigenproblems

Paul Binding
University of Calgary

Many apparently non-self-adjoint eigenproblems can be recast in the form $Ay - \lambda By = 0$ where A and B are self-adjoint operators in a suitable Hilbert space. They may however be indefinite, and the use of a two parameter embedding $Ay - \lambda By = \mu y$ will be explored. Under fairly general conditions this allows one to "see" certain aspects of the spectrum in terms of the (λ, μ) eigencurves.

2.9.5 Characterization of Input for Livšic systems

Grant M. Boquet
Metron Corporation, Reston, VA

In the construction of the governing equations for two-dimensional discrete-time Livšic systems one encounters **compatibility condition** pencil of the form

$$\sigma_2 u(t_1 + 1, t_2) - \sigma_1 u(t_1, t_2 + 1) + \gamma u(t_1, t_2) = 0 \qquad (2.1)$$

where σ_1, σ_2 and γ are constant matrices mapping the input to an ambient input space that, later on, is identified with the state space. Alternatively, one may reach (2.1) as a **determinantal representation**. From (1) we observe that the input trajectories for a vessel may not be arbitrarily specified along the entire time domain $\mathbb{N}^2$; however, we may freely specify the **boundary conditions** of $u(t_1, t_2)$ along the axis lines $\{(0, t_2) : t_2 \in \mathbb{N}\}$ or $\{(t_1, 0) : t_1 \in \mathbb{N}\}$.

We first provide an explicit form of the solutions to (2.1) in terms of boundary conditions for particular conditions on σ_1, σ_2 and γ. In the scalar case these are functions of the form

$$u(t_1, t_2) = \left(\frac{\gamma}{\sigma_1}\right)^{t_2} \sum_{i=0}^{t_2} \binom{t_2}{i} \left(\frac{\sigma_2}{\gamma}\right)^{i} u(i + t_1, 0).$$

Since the boundary conditions may be freely specified along the domain of a one-dimensional system, we consider coupling the boundary conditions to an additional constraint via a system of **time-dependent** difference equations of the form

$$\begin{cases} \sigma_2 u(t_1 + 1, t_2) - \sigma_1 u(t_1, t_2 + 1) + \gamma u(t_1, t_2) & = 0 \\ \sigma_2 u(t_1 + 1, 0) + \alpha(t_1) u(t_1, 0) & = 0 \end{cases} .$$

In conclusion we consider these "coupling extensions" for a selection of **multivariate extensions of special functions** (defined via time-dependent recurrence relations) and **synthetic aperture sonar** (SAS).

2.9.6 Generalized repeated interaction model and transfer functions

Santanu Dey
Indian Institute of Technology, Bombay

Using a scheme involving a lifting of a row contraction we introduce a toy model of repeated interactions between quantum systems. In this model there is an outgoing Cuntz scattering system involving two wandering subspaces. We associate to this model an input/output linear system which leads to a transfer function. This transfer function is a multi-analytic operator, and we show that it is inner if we assume that the system is observable. Finally it is established that transfer functions coincide with characteristic functions of associated liftings.

2.9.7 A hierarchy of Von Neumann Inequalities?

Quanlei Fang
CUNY-BCC

The well-known von Neumann inequality for commuting row contractions can be interpreted as saying that the tuple

$$(M_{z_1}, \dots, M_{z_n})$$

on the Drury-Arveson space H_n^2 dominates every other commuting row contraction $(A_1, \dots, A_n)$. We show that a similar domination relation exists among certain pairs of “lesser” row contractions $(A_1, \dots, A_n)$ and $(B_1, \dots, B_n)$. It hints at a possible hierarchical structure among the family of commuting row contractions. This is a joint work with Jingbo Xia.

2.9.8 Semi-algebraic geometry designed for matrix variables

Bill Helton
UC San Diego

Bill’s talks at IWOTA will describe the recent development of free analogs of two different classical subjects, real algebraic geometry and convex optimization. This one focuses on recent developments in non-commutative real algebraic geometry. It will be co-ordinated with Igor Klep's plenary talk.

The classic branch of real algebraic geometry, also called semi-algebraic geometry, analyzes why polynomial inequalities hold. For example, a polynomial p being positive where the polynomial q is positive is closely related to the algebraic relation $p = \Sigma^2 + \Sigma^2 q$, where Σ^2 stands for a sum of squares of polynomials. Results which give an algebraic explanation for positivity of a polynomial on a given semi-algebraic set are called Positivstellensätze.

Free semi-algebraic geometry is the study of (matrix-valued) polynomial inequalities in freely non-commuting variables. A given polynomial p in g freely non-commuting variables is naturally evaluated at a g-tuple X of symmetric matrices (of the same size n) yielding a matrix $p(X)$ and p is symmetric if $p(X)$ is symmetric for all such X. An example of a matrix (or free) inequality is, for free symmetric polynomials p and q,

$$p(X) \succ 0 \text{ for all } X \text{ such that } q(X) \succ 0,$$

where $A \succ 0$ means the symmetric matrix A is positive definite.

Free semi-algebraic geometry has emerged in the last 10 years to analyze such inequalities. Indeed many Positivstellensätze in the free setting have cleaner statements than do their commutative counterparts and there are now a number of appealing free Real Nullstellensätze.

The talk describes recent results obtained jointly with Igor Klep, Scott McCullough, Harry Dym, Damon Hay, Chris Nelson, Nick Slingeland and Victor Vinnikov.

2.9.9 The Bessmertnyi class: old and new results

Dmitry Kaliuzhnyi-Verbovetskyi
Drexel University

In the early 1980s, M. F. Bessmertnyi introduced a class of matrix-valued rational functions of d variables which admit a so-called finite-dimensional long resolvent representation. The motivation came from electrical engineering, namely, this is the class of impedance functions of passive electrical $2n$ ports where impedances of elements (resistances, capacitances, inductances) are considered as independent variables. Bessmertnyi gave several necessary conditions for a function to be in this class, however no good characterization of the class in intrinsic terms (as opposed to 'existence of a representation' terms) was given. Later on, in my 2004 paper I extended the definition of Bessmertnyi's class to not necessarily rational, operator-valued functions, which admit an infinite-dimensional long resolvent representation, and gave three characterizations of the extended class, one of which was in intrinsic terms. Also, the connection to the Schur–Agler class of analytic functions on the unit polydisk was established. In my recent work with J. A. Ball, characterizations of the original Bessmertnyi's class have been obtained, including an intrinsic one.

2.9.10 Free Convexity and Changes of Variables

Igor Klep
The University of Auckland

The talk treats several topics involving *free analytic maps*. These maps are free analogs of classical analytic functions in several complex variables, and are defined in terms of non-commuting variables amongst which there are no relations - they are *free variables*. Analytic free maps include vector-valued polynomials in free (non-commuting) variables and form a canonical class of mappings from one noncommutative domain $\mathcal{D}$ in say g variables to another noncommutative domain $\tilde{\mathcal{D}}$ in $\tilde{g}$ variables.

Analytic free maps tend to be very rigid. Here is a sample. As a natural extension of the usual notion, an analytic free map is *proper* if it maps the boundary of $\mathcal{D}$ into the boundary of $\tilde{\mathcal{D}}$. Roughly speaking, proper analytic free maps are one-to-one, and if $g = \tilde{g}$, then f is invertible and f^{-1} is also an analytic free map.

We will focus our attention to *convexity* and investigate which noncommutative domains $\mathcal{D}$ can be analytically transformed into convex noncommutative domains. Of particular interest are convex domains obtained from linear matrix inequalities (LMIs).

The talk will describe the status of recent advances on this. The work is joint with Bill Helton and Scott McCullough, so the talk we be coordinated with Bill Helton's talk in the same session.

2.9.11 Positivity, Sums of Squares and the Multi-Dimensional Moment Problem

Salma Kuhlmann
University of Konstanz

In this talk, I will explain the relationship between the multi-dimensional moment problem on the one hand, and sums of squares of polynomials on the other. I will give a quick survey of various Positivstellensätze in Real Algebraic Geometry, and show how they apply to representation of continuous linear functionals (on the polynomial algebra) by positive Borel measures.

2.9.12 Properties of complex symmetric operators

Ji Eun Lee
Institute of Mathematical Sciences, Ewha Womans University, Korea

We say that an operator $T \in \mathcal{L}(H)$ is complex symmetric if there exists a conjugation C on $\mathcal{H}$ such that $T = CT^*C$. In this paper, we prove that every complex symmetric operator $T \in \mathcal{L}(H)$ is quasitriangular. In particular, we consider connection between complex symmetric operators and their adjoints in the context of power regularity, cyclicity, weak hypercyclicity, (weak) supercyclicity, hyperinvariant subspaces. We establish several equivalences for a complex symmetric operator of generalized Weyl's, a-Weyl's, generalized Browder's, Browder's, generalized a-Browder's theorem, and a-Browder's theorem, respectively.

2.9.13 Dilation theory in finite dimensions

Orr Shalit
Department of Mathematics, Ben-Gurion University of the Negev

Traditionally, "dilation theory" belongs to the realm of operator theory on infinite dimensional spaces. Is there an effective version of dilation theory that does not require infinite dimensional spaces? If so, does this

theory teach us something new? I hope to convince you that the answer to both questions is “yes”.

For example, I will present the following finite dimensional analogue of Ando’s Theorem.

Theorem. *Let A and B be two commuting contractions on a finite dimensional Hilbert space H. Then for every N there exists two commuting unitaries V, U on a finite dimensional space $K \supseteq H$ such that for all $m, n \leq N$,*

$$A^m B^n = P_H V^m U^n P_H.$$

The talk is based on joint works with Eliahu Levy and John McCarthy.

[1] E. Levy and O. M. Shalit. Dilation theory in finite dimensions: the possible, the impossible and the unknown. *Rocky Mountain J. Math.*, to appear.

[2] J. E. McCarthy and O. M. Shalit. Unitary N-dilations for tuples of commuting matrices. *Proc. Amer. Math. Soc.*, to appear.

2.9.14 Commutative Livsic Overdetermined Three-Dimensional Linear Systems

Eli Shamovich

Ben-Gurion University of the Negev

The study of linear input/output overdetermined systems was initiated by M. Livsic and his co-workers (cf. [1]). The commutative two-dimensional case was thoroughly studied in [1], [2] and [3]. Since the system is overdetermined it requires compatibility conditions. It turns out that the system of compatibility conditions in the three-dimensional case is itself overdetermined. We will introduce the additional necessary and sufficient compatibility conditions required and show how those conditions translate in the frequency domain into geometric requirements. This talk is based on joint work with Victor Vinnikov.

[1] Livšic, M. S. and Kravitsky, N. and Markus, A. S. and Vinnikov, V. Theory of commuting nonselfadjoint operators *volume 332 of Mathematics and its Applications*. Kluwer Academic Publishers Group. Dodrecht (1995).

[2] Ball, Joseph A. and Vinnikov, Victor. Overdetermined multidimensional systems: state space and frequency domain methods, *volume 134 of Mathematical systems theory in biology, communications, computation, and finance*. IMA Vol. Math. Appl., Springer. New-York (2003), pp. 63-119.

[3] Vinnikov, Victor. Commuting operators and function theory on a Riemann surface, *volume 33 of Holomorphic spaces (Berkeley, CA, 1995)*. Math. Sci. Res. Inst. Publ., Cambridge Univ. Press. Cambridge (1998), pp. 445-476.

2.9.15 Hardy classes on noncommutative domains

Victor Vinnikov
Ben Gurion University of the Negev

For a variety of finite-dimensional operator spaces, we study the Hardy space H^2 of locally bounded (or equivalently analytic) noncommutative functions on the noncommutative unit ball. Here the noncommutative unit ball is the disjoint union of unit balls of square matrices of all sizes over the original vector space and a noncommutative function is a function that respects direct sums and similarities. Our tools combine the general theory of noncommutative functions developed recently by the speaker and D. S. Kaliuzhnyi-Verbovetskyi with the results about asymptotic freeness originating with D.-V. Voiculscu. This is a joint work with Mihai Popa.

2.9.16 Nevanlinna representations in several variables

Nicholas Young
Department of Pure Mathematics, Leeds University and School of Mathematics and Statistics, Newcastle University

We generalize two integral representation formulae of Nevanlinna to functions of several variables. We show that for a large class of analytic functions that have non-negative imaginary part on the upper polyhalfplane there are representation formulae in terms of densely defined self-adjoint operators on a Hilbert space. We introduce three types of structured resolvent of a self-adjoint operator and identify four different types of representation in terms of these resolvents. We relate the types of representation that a function admits to its growth at infinity.

2.10 Special session on Dynamics and Operator Algebras

Speakers

2.10.1 Simplicity of groupoid $C*$-algebras

Jonathan Brown
University of Otago

In joint work with Lisa Clark, Cythia Farthing and Aidan Sims, we examine the C^*-algebras $C^*(G)$ associated to locally compact Hausdorff, étale groupoids G. We also show that if G is second-countable, then the interior of the isotropy of G is just the unit space if and only if G is topologically principal in Renault's sense. We use this to prove that if G is second-countable and its full and reduced C^*-algebras coincide, then $C^*(G)$ is simple if and only if G is topologically principal and minimal. This result recovers the existing simplicity results for k-graphs and Deaconu-Renault groupoids.

2.10.2 The equivalence relations of quotient maps

Lisa Clark
University of Otago

In this talk, I will discuss the groupoid C^*-algebras $C^*(R(\psi))$ associated to the equivalence relation $R(\psi)$ induced by a quotient map $\psi : Y \to X$. If Y is Hausdorff then $C^*(R(\psi))$ is a Fell algebra, and if both Y and X are Hausdorff then $C^*(R(\psi))$ has continuous trace. Astrid an Huef, Iain Raeburn and I show that the C^*-algebra $C^*(G)$ of a locally compact, Hausdorff and principal groupoid G is a Fell algebra if and only if G is topologically isomorphic to some $R(\psi)$, extending a theorem of Archbold and Somerset.

2.10.3 Unital Dilations of Completely Positive Semigroups

Dave Gaebler
University of Iowa

Semigroups of completely positive maps on C*-algebras arise in the dynamics of open quantum systems, and in the theory of noncommutative Markov processes. Several authors have studied how such a semigroup may be dilated to a semigroup of endomorphisms; however, the dilations achieved are generally non-unital, corresponding to the embedding of $B(H)$ as a corner of $B(K)$ for Hilbert spaces $H \subset K$. Jean-Luc Sauvageot's dilation theorem, published in 1986, achieves a unital dilation, but at the cost of important continuity properties. This talk will discuss Sauvageot's approach to dilation and its relationship to free probability and other subsequent developments.

2.10.4 Toeplitz index theory and K-theory

Jerry Kaminker
University of California Davis

We will discuss various ways in which the index of Toeplitz operators can be expressed in terms of the index of elliptic differential operators. This has some actual and potential applications to problems involving contact manifolds. This is joint work with Ron Douglas and Xiang Tang.

2.10.5 Characterization of the primitive ideal space of row-finite k-graphs

Sooran Kang
University of Wollongong

The primitive ideal space of the C^*-algebras for directed graphs was described by Raeburn et al in terms of the structural properties of the directed graphs. For k-graphs, Sims produced a complete description of the lattice of gauge-invariant ideals in C^*-algebras for finitely aligned k-graphs. By extending the methods developed for directed graphs and using the results of Sims, we describe the gauge-invariant primitive ideals of aperiodic k-graphs. Also, we characterize the primitive ideal spaces of the C^*-algebras for strongly aperiodic k-graphs by certain topological spaces.

This is joint work with David Pask of the University of Wollongong, Australia.

2.10.6 Ideals and quotients of labeled graph C^*-algebras

Sun Ho Kim

In this talk, we consider the gauge-invariant ideal structure of a C^*-algebra $C^*(E, \mathcal{L}, \mathcal{B})$ associated to a labeled space $(E, \mathcal{L}, \mathcal{B})$. Under the assumption that an accommodating set $\mathcal{B}$ is closed under relative complements, it is obtained that there is a one to one correspondence between the set of all hereditary saturated subsets of $\mathcal{B}$ and the gauge-invariant ideals of $C^*(E, \mathcal{L}, \mathcal{B})$. For this, we introduce a quotient labeled space $(E, \mathcal{L}, [\mathcal{B}]_{\mathrm{R}})$ arising from an equivalence relation $\sim_{\mathrm{R}}$ on $\mathcal{B}$ and show the existence of the C^*-algebra $C^*(E, \mathcal{L}, [\mathcal{B}]_{\mathrm{R}})$ generated by a universal representation of $(E, \mathcal{L}, [\mathcal{B}]_{\mathrm{R}})$. Also the gauge-invariant uniqueness theorem for $C^*(E, \mathcal{L}, [\mathcal{B}]_{\mathrm{R}})$ is obtained. Finally we find a condition for a quotient labeled space for which the corresponding quotient labeled graph C^*-algebra can be realized as a labeled graph C^*-algebra.

This is joint work with Ja A Jeong of Seoul National University and Gi Hyun Park of Hanshin University.

2.10.7 Quantum Heisenberg manifolds as twisted groupoid C^*-algebras

Alex Kumjian
University of Nevada

Quantum Heisenberg manifolds were introduced by Rieffel more than 20 years ago as examples illustrating the phenomenon of deformation quantization. They have been the subject of much interest ever since. Contributors to research in this area include Kang, Abadie, Eilers and Exel. The last three authors proved that each such C*-algebra is isomorphic to a crossed product of the continuous functions on a two torus by a Hilbert C^*-bimodule. Abadie and Exel next observed that the Hilbert C^*-bimodule is determined by a homeomorphism of the torus and an element of the classical Picard group. Using work of Deaconu, Kumjian and Muhly, we show that each Quantum Heisenberg manifold is isomorphic to a twisted groupoid C^*-algebra where the groupoid is the transformation groupoid for the homeomorphism and the twist arises from the element of the classical Picard group.

This talk is based on joint work with Sooran Kang.

2.10.8 Non-commutative solenoids

Judith Packer
University of Colorado at Boulder

A noncommutative solenoid is a twisted group C*-algebra for $\mathbb{Q}_N \times \mathbb{Q}_N$, where $\mathbb{Q}_N$ denotes the N-adic rational numbers. In this talk, we discuss techniques for determining when these C*-algebras are simple, and classify a family of these C*-algebras are simple, and classify a family of these C*-algebras up to *-isomorphism through studying the multipliers of $\mathbb{Q}_N \times \mathbb{Q}_N$. Our work relies in part on computing the K_0 groups of the C*-algebras in question. We do so by writing these C*-algebras as direct limits of rotation algebras, and we prove that the K_0 groups are extensions of $\mathbb{Q}_N$ by $\mathbb{Z}$.

This work is joint with Frédéric Latrémolière of the University of Denver.

2.10.9 The K-theory of labelled graph C*-algebras

David Pask
University of Wollongong

I will discuss joint work with Toke Carlsen and Teresa Bates in which we give a formula for, and compute the K-theory of labelled graph C*-algebras.

2.10.10 Coactions on Cuntz-Pimsner algebras

Dave Robertson
University of Wollongong

Given a C*-correspondence X over a C*-algebra A, there is a notion of the multiplier correspondence $M(X)$, which has natural structure as a C^*-correspondence over the multiplier algebra $M(A)$. This concept allows us to define a coaction of a locally compact group G on a C^*-correspondence X. I will show that given a suitable covariance condition, such a coaction lifts to a coaction of G on the Cuntz-Pimsner algebra $\mathcal{O}_X$.

2.10.11 When is the C*-algebra of a higher-rank graph approximately finite-dimensional?

Aidan Sims
University of Wollongong

The C^*-algebra of a directed graph is approximately finite-dimensional if and only if the graph has no cycles. The proof of this result, first obtained by Kumjian, Pask and Raeburn in one of the first papers on the subject, has by now been refined to the point where it is almost elementary. By contrast, the question of when the C^*-algebra of a higher-rank graph is approximately finite-dimensional is still open twelve years after these C*-algebras first appeared. I will discuss some recent progress on this question and indicate what makes it so much harder for higher-rank graphs than for ordinary graphs. This is joint work with D. Gwion Evans.

2.10.12 Derivations in the ideals of compact operators

Fedor Sukochev
University of New South Wales

Let $\mathcal{I}, \mathcal{J}$ be symmetric quasi-Banach ideals of compact operators on an infinite-dimensional complex Hilbert space H, let $\mathcal{J} : \mathcal{I}$ be a space of multipliers from $\mathcal{I}$ to $\mathcal{J}$. Obviously, ideals $\mathcal{I}$ and $\mathcal{J}$ are quasi-Banach algebras and it is clear that any ideal $\mathcal{J}$ is a bimodule for $\mathcal{I}$. We study the set of all derivations from $\mathcal{I}$ into $\mathcal{J}$. We show that any such derivation is automatically continuous and there exists an operator $a \in \mathcal{J} : \mathcal{I}$ such that $\delta(\cdot) = [a, \cdot]$, moreover $\|\delta\|_{\mathcal{B}(\mathcal{I},\mathcal{J})} \leqslant 2C\|a\|_{\mathcal{J}:\mathcal{I}}$, where C is a modulus of concavity of the quasi-norm $\|\cdot\|_{\mathcal{J}}$. In the special case, when $\mathcal{I} = \mathcal{J} = \mathcal{K}(H)$ is a symmetric Banach ideal of compact operators on H, our result yields the classical fact that any derivation δ on $\mathcal{K}(H)$ may be written as $\delta(\cdot) = [a, \cdot]$, where a is some bounded operator on H and $\|\delta\|_{\mathcal{B}(\mathcal{I},\mathcal{J})} \leqslant 2\|a\|_{\mathcal{B}(H)}$.

2.10.13 From k-coloured graphs to k-graphs

Samuel Webster
University of Wollongong

Kumjian and Pask defined k-graphs as a models for the higher-rank

Cuntz-Krieger algebras introduced by Robertson and Steger. Higher-rank graphs are higher-dimensional analogues of directed-graphs, but are defined using the language of category theory. In this paper we provide a detailed and constructive description of a higher-rank graph in the more familiar language of directed graphs, and characterise exactly when a k-coloured graph gives rise to a k-graph.

2.10.14 KMS states for self-similar actions

Michael Whittaker
University of Wollongong

A self-similar action (G, X) consists of a group G along with a self-similar action of the group on a rooted tree. Heuristically, self-similarity is displayed when the action of the group repeats at all levels of the tree, in a similar fashion to fractals where patterns are repeated at all scales. Self-similar actions give rise to Cuntz-Pimsner algebras, discovered by Nekrashevych. We describe KMS states on these algebras. Several examples will be presented.

This is joint work with Marcelo Laca, Iain Raeburn, and Jacqui Ramagge.

2.10.15 Coisometric Extensions of Completely Contractive Covariant Representations

Travis Wolf
University of Iowa

Every C*-correspondence E over a C*-algebra A comes equipped with a homomorphism $\phi : A \to \mathcal{L}(E)$ giving the left action. A *generalized transfer operator* for ϕ is a completely positive linear map $\tau : \mathcal{L}(E) \to A$ with the property that $\tau(\phi(a)X) = a\tau(X)$ for $a \in A$ and $X \in \mathcal{L}(E)$. Assuming ϕ has a generalized transfer operator, we give a way of constructing a unique coisometric extension for any completely contractive covariant representation of (E, A) on a Hilbert space. Completely contractive covariant representations are generalizations of contractive operators to the setting of C^*-correspondences, and in the course of the construction we will examine several examples. Notable among them is the special case where $E = \mathbb{C}^d$ and $A = \mathbb{C}$, in which case a (unital) generalized transfer operator is simply a state on $M_d(\mathbb{C})$.